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SEAT No._____

No. of Printed Pages: 3

Sardar Patel University
B.Sc. Semester-III Examination - 2022
Friday, 18th November, 2022
Course Code: US 03CSTA52
(Elements of Probability Theory)

Time: 10 to 1 p.m

M.Marks: 70

Note: (i) Simple/Scientific calculator is allowed   (ii) Q.3 to 6 each sub questions have 5 marks.

Q.1 Multiple Choice Questions   (10 × 1)

(1)   Let X be a r.v. with probability distribution
        $f(x) = 2/3^x, x = 1, 2, ...$ and zero otherwise, then probability that X is even
        (a) 1/4   (b) 1/2   (c) 1/36   (d) 1/8

(2)   Given that $V(X) = 4$ then $V(Y) = $ _____ where $Y = (X - 2)/3$
        (a) 2/3   (b) −14/9   (c) 4/9   (d) 4/3

(3) $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then
        (a) $P(B/A) = P(B) - P(A)$ (b) $P(A' \cup B') = P(A') + P(B')$
        (c) $P(A \cup B') = P(A')P(B')$ (d) $P(A/B) = P(A)$

(4)   If $f(x) = kx, 0 < x < 2$
        $= k(5 - x), 2 \leq x < 5$
        $= 0$, otherwise, is the p.d.f. of a r.v. X then the value of $k = $ _____
        (a) 0   (b) 1/13   (c) 2/13   (d) 3/13

(5)   Given that $f(x) = k(1 + x)^2, x = 1, 2, 3$ and zero otherwise. For what value of k given $f(x)$ being p.m.f?
        (a) 7/30   (b) 1/29   (c) 1/30   (d) 7/29

(6)   The p.g.f. of a r.v. X is $P(t) = 1/(1 - t/3)$ then $P(X = 2)$ is
        (a) $(1/3)^3$ (b) $(1/3)^2$ (c) 1/3   (d) 1

(7)   If A and B are two events such that $B \subset A, P(B) \neq 0$, then which of the following is correct?
        (a) $P(B/A) = P(B)/P(A)$ (b) $P(A/B) = P(A)/P(B)$ (c) $P(B/A) = 1$ (d) None of these

(8)   If $f(x, y) = (x^2 + y^2)/112, x, y = 0, 1, 2, 3$, is the joint probability distribution of X and Y then $P(Y = 2) = $ _____
        (a) 14/112   (b) 18/112   (c) 30/112   (d) 50/112

(9)   If $P(A) = 0.7, P(B) = 0.2, P(A^c \cup B^c) = 0.9$, then prob. that at least one of the event occurs is
        (a) 0.9   (b) 0.5   (c) 0.2   (d) 0.8

(10)   Let X and Y have the joint pdf $f(x, y) = \left(\frac{4}{3}\right)(1 - xy), 0 < (x, y) < 1$ and zero otherwise, then $E(X) = $ _____
        (a) 9/4   (b) 13/162   (c) 4/9   (d) None of these

Q.2   Short Type Questions (Attempt Any Ten)   (10 × 2)

(1)   Given that $f(x) = k(1/2)^x$, is a probability distribution of a r.v. X which takes on values $0, 1, 2, ..., 6$.
        Find the value of k and $P(X \leq x)$.

(2)   Find μ_2' if $f(x) = \frac{3}{10}(3x - x^2), 0 < x < 2$ and zero otherwise

(3)   For two events A and B, if $A \subset B$ then $P(A/B) = $ _____
        Fill in the blank and show the same by giving counter example.

(4)   Determine $V(X)$ if (i) $M_X(t) = 1/(1 - 5t)$ (ii) $M_X(t) = 0.7 + 0.3e^t$

(5)   The joint probability distribution of X and Y is as follows:

		Y	
X	−1	0	1
−1	0	0.2	0
0	0.2	0.2	0.2
1	0	0.2	0

        Find the $Cov(Y, Y)$

(6)   Evaluate $P(A \cup B)$ if $2P(A) = P(B) = 5/13$ and $P(A/B) = 2/5$

(7)   Check whether the following function is p.m.f. or not?

        $f(x) = \frac{1}{2x}, x = 1, 2, ...$ and zero otherwise

(8) Find $E(X)$ if the distribution function of X is

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1/2 & , 0 \le x < 1 \\ 5/6 & , 1 \le x < 2 \\ 1 & , x \ge 2 \end{cases}$$

(9) If $f(x) = 1/4, -2 < x < 2$ *and zero oterwise*, is the pdf of X. Show that all the odd order moments are zero.

(10) The joint p.d.f of X and Y is $f(x,y) = k(x + 3y), 0 < x < 1, 0 < y < 1$ *and zero otherwise*

(i) Determine the value of k (ii) the marginal distribution of X.

(11) If $P(t) = (3 - 2t)^{-1}$ is the p.g.f. of a r.v. X then find $P(X \ge 1)$,

(12) State the law of total probability

Q.3(a) State and prove law of addition for the probabilities of three events.

(b) Following are the compositions of two basket flowers:

Basket - I	Basket - II
Pink - 5	Pink - 4
White – 3	White – 5
Yellow – 2	Yellow – 4

One basket is chosen at random and two flowers drawn from it they happen to be white and yellow. What is the probability that they come from basket – I, II?

OR

Q.3(a) Do as directed :

(i) If $A \subset B$ then prove that $P(A) \le P(B)$ (ii) $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$

(b) A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 of sales department and 1 chartered accountant. Find the prob. of forming the committee in the following manners:

(i) There must be one from each category (ii) It should have at least one from purchase department (iii) chartered accountant must be in the committee.

Q.4(a) A lot contains 10 items of which 3 are defective. Three items are drawn at random one after other without replacement. Find the probability distribution of no. of non – defective items selected. Determine (i) $P(X < 1)$ (ii) $P(X \ge 2)$

(b) Let X be a r.v. with p.d.f. given as

$$f(x) = \begin{cases} kx & , 0 \le x < 1 \\ = k & , 1 \le x < 2 \\ = 3k - kx & , 2 \le x < 3 \\ = 0 & , \text{otherwise} \end{cases}$$

(i) Determine k (ii) the c.d.f. of X (iii) $P(X < 1/2)$ (iv) $P(X > 1/3)$ (v) $P(1/2 < X < 1)$

OR

Q.4(a) A coin is tossed until a head comes up (occur) for the first time. Describe the sample space. Is X discrete r.v or not? Find the probability distribution of X, where X is no. of tails before a head.

(b) Given the p.d.f

$f(x) = ke^{-4x}, x > 0$ and zero otherwise (i) Determine the value of k (ii) the c.d.f. of X

(iii) Evaluate $P(X > 3), P(2X + 3 > 5)$ and $P(2 < X < 4)$.

Q.5(a) The probability distribution of a r.v. X is

$P(X = x) = (3/4)^x (1/4), x = 0, 1, 2, \ldots$ *and zero otherwise*

Find (i) the mean and variance of X (ii) $P(X \text{ is even})$

(b) If $f(x) = 1/10, -5 < x < 5$ *and zero otherwise*, is the pdf of X.

Find the m.g.f of X and hence mean and variance of X.

OR

Q.5(a) If $f(x) = \dfrac{e^{-1}}{x!}, x = 0, 1, 2, \ldots$ *and zero otherwise*, is the p.m.f. of X. Find (i) m.g.f (ii) c.g.f (iii) β_1, β_2

(b) Two balls drawn with replacement from a box containing 8 red and 4 white balls. If X denotes the no. of white balls drawn. Find the probability distribution of X. Find the mean and variance of X.

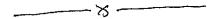**Q.6** Consider two r.v's X and Y with joint probability distribution given in the following table:

X	Y		
	2	4	5
1	1/12	1/24	1/24
2	1/6	1/12	1/8
3	1/4	1/8	1/12

Find $(i) P(X \le 2, Y \le 4)$ (ii) $P(Y = 2/X = 1)$ (iii) the marginal distribution of X and Y (iv) Are X and Y independent? (v) the correlation coefficient (vi) the conditional distribution of Y given $X = 2$.

OR

Q.6(a) Do as directed:

(i) Prove that two independent r.v's are uncorrelated but converse is not true.

(ii) The joint p.d.f. of two r.v's X and Y is given by $f(x,y) = C(2x + y), 2 < x < 6; 0 < y < 5$ and zero otherwise

Find (a) the constant C (b) the marginal distribution of X (c) $P(X > 3, Y > 2)$

(b) Given $f(x,y) = k(x + 2y), x, y = 1, 2$ and zero otherwise, is the joint p.m.f. of X and Y.

Find (i) the constant k (ii) $P(X \ge 1, Y \le 1)$ (iii) $P(X + Y \le 2)$ (iv) $P(X < Y)$.

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