



Sardar Patel University
 B.Sc. Semester-III Examination - 2022
 Friday, 18th November, 2022
 Course Code: US-03CSTA22
 (Elements of Probability Theory)

Time: 10 to 1 p.m.

M.Marks: 70

Note: (i) Simple/Scientific calculator is allowed (ii) Q.3 to 6 each sub questions have 5 marks.

Q.1 Multiple Choice Questions

(10 × 1)

- (1) Two balls are drawn at random without replacement from a box containing 6 red and 10 black balls. The probability that first ball is black and second is red _____
 (a) 64/15 (b) 1/8 (c) 1/2 (d) 15/64
- (2) A r.v. X has p.d.f. $f(x) = 1/4, 0 < x < 4$ and zero otherwise, then $P(|X| > 1) =$ _____
 (a) 1/2 (b) 1 (c) 2/3 (d) 3/4
- (3) If $M_x(t) = e^{2(e^t-1)}$ is the m.g.f of a random variable X then mean = _____
 (a) 1 (b) 2 (c) 3 (d) 4
- (4) Which of the following is not true?
 (a) $E(X) = \left[\frac{d}{dt} M_x(t) \right]_{t=0}$ (b) $\beta_1 = \frac{\mu_4}{\mu_2^2}$
 (c) $M_x(t) = P_x(e^t)$ (d) $Cov(X, X) = V(X)$
- (5) Of 10 girls in a class, 4 have knowledge of R software. If three of the girls are chosen at random, the probability that all of them have knowledge of R software is _____
 (a) 3/10 (b) 1/120 (c) 121/120 (d) 1/30
- (6) The joint p.d.f of X and Y is $f(x, y) = k(x + 3y), 0 < x < 1, 0 < y < 1$ and zero otherwise then the value of $k =$ _____
 (a) 1/2 (b) 1/3 (c) 1/4 (d) 1/8
- (7) Let A, B and C be independent events with $P(A) = 0.5, P(B) = 0.6$ and $P(C) = 0.1$ then $P(A^c \cup B^c \cup C^c)$ is
 (a) 0.69 (b) 0.71 (c) 0.73 (d) 0.97
- (8) Given that $f(x) = k(1+x)^2, x = 1, 2, 3$ and zero otherwise. What will be the value of k so that given $f(x)$ being p.m.f?
 (a) 7/30 (b) 1/30 (c) 1/29 (d) 4/29
- (9) If A and B are independent events then which of the following statements are true?
 (a) A and B^c are independent (b) A^c and B^c are independent
 (c) A^c and B are independent (d) All of the above
- (10) Let X and Y have the joint pdf $f(x, y) = \left(\frac{4}{3}\right)(1 - xy), 0 < (x, y) < 1$ and zero otherwise, then $E(X) =$ _____
 (a) 9/4 (b) 13/162 (c) 4/9 (d) None of these

(10 × 2)

Q.2 Short Type Questions (Attempt Any Ten)

- (1) Given that $f(x) = k(1/2)^x$, is a probability distribution of a r.v. X which takes on values $0, 1, 2, \dots, 6$. Find k and $P(X \leq x)$.
- (2) Find μ_2 if $f(x) = \frac{3}{10}(3x - x^2), 0 < x < 2$ and zero otherwise
- (3) For two events A and B , if $A \subset B$ then $P(A/B) =$ _____
 Fill in the blank and show the same by giving counter example.
- (4) Determine $V(X)$ if (i) $M_x(t) = 1/(1 - 5t)$ (ii) $M_x(t) = 0.7 + 0.3e^t$
- (5) The joint probability distribution of X and Y is as follows:

X	Y		
	-1	0	1
-1	0	0.2	0
0	0.2	0.2	0.2
1	0	0.2	0

Find the $Cov(Y, Y)$

- (6) Evaluate $P(A \cup B)$ if $2P(A) = P(B) = 5/13$ and $P(A/B) = 2/5$

- (7) Check whether the following function is p.m.f. or not?

$$f(x) = \frac{1}{2x}, x = 1, 2, \dots \text{ and zero otherwise}$$

- (8) Find $E(X)$ if the distribution function of X is

$$F(x) = \begin{cases} 0 & , x < 0 \\ 1/2 & , 0 \leq x < 1 \\ 5/6 & , 1 \leq x < 2 \\ 1 & , x \geq 2 \end{cases}$$

- (9) If $f(x) = 1/4, -2 < x < 2$ and zero otherwise, is the pdf of X . Show that all the odd order moments are zero.

- (10) The joint p.d.f of X and Y is $f(x, y) = k(x + 3y), 0 < x < 1, 0 < y < 1$ and zero otherwise

(i) Determine the value of k (ii) the marginal distribution of X

- (11) If $P(t) = (3 - 2t)^{-1}$ is the p.g.f. of a r.v. X then find $P(X \geq 1)$,

- (12) State the law of total probability.

- Q.3(a) State and prove law of addition for the probabilities of three events.

- (b) Following are the compositions of two basket flowers:

Basket - I	Basket - II
Pink - 5	Pink - 4
White - 3	White - 5
Yellow - 2	Yellow - 4

One basket is chosen at random and two flowers drawn from it they happen to be white and yellow. What is the probability that they come from basket - I, II?

OR

- Q.3(a) Do as directed :

(i) If $A \subset B$ then prove that $P(A) \leq P(B)$ (ii) $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

- (b) A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 of sales department and 1 chartered accountant. Find the prob. of forming the committee in the following manners:

(i) There must be one from each category (ii) It should have at least one from purchase department (iii) chartered accountant must be in the committee.

- Q.4(a) A lot contains 10 items of which 3 are defective. Two items are drawn at random one after other without replacement. Find the probability distribution of no. of non-defective items selected.

- (b) Let X be a r.v. with p.d.f. given as

$$f(x) = \begin{cases} kx & , 0 \leq x < 1 \\ k & , 1 \leq x < 2 \\ 3k - kx & , 2 \leq x < 3 \\ 0 & , \text{otherwise} \end{cases}$$

(i) Determine k (ii) the c.d.f. of X (iii) $P(X < 1/2)$ (iv) $P(X > 1/3)$ (v) $P(1/2 < X < 1)$

OR

- Q.4(a) A coin is tossed until a head comes up (occur) for the first time. Describe the sample space. Is X discrete r.v or not? Find the probability distribution of X , where X is no. of tails before a head.

- (b) Given the p.d.f

$f(x) = ke^{-4x}, x > 0$ and zero otherwise (i) Determine the value of k (ii) the c.d.f. of X

(iii) Evaluate $P(X > 3), P(2X + 3 > 5)$ and $P(2 < X < 4)$.

- Q.5(a) The probability distribution of a r.v. X is

$$P(X = x) = (3/4)^x(1/4), x = 0, 1, 2, \dots \text{ and zero otherwise}$$

Find (i) the mean and variance of X (ii) $P(X \text{ is even})$

- (b) If $f(x) = 1/10, -5 < x < 5$ and zero otherwise, is the pdf of X .

Find the m.g.f of X and hence mean and variance of X .

OR

- Q.5(a) If $f(x) = \frac{e^{-1}}{x!}$, $x = 0, 1, 2, \dots$ and zero otherwise, is the p.m.f. of X . Find (i) m.g.f (ii) c.g.f (iii) β_1, β_2
- (b) Two balls drawn with replacement from a box containing 8 red and 4 white balls. If X denotes the no. of white balls drawn. Find the probability distribution of X . Find the mean and variance of X .
- Q.6 Consider two r.v's X and Y with joint probability distribution given in the following table:

X	Y		
	2	4	5
1	1/12	1/24	1/24
2	1/6	1/12	1/8
3	1/4	1/8	1/12

Find (i) $P(X \leq 2, Y \leq 4)$ (ii) $P(Y = 2/X = 1)$ (iii) the marginal distribution of X and Y (iv) Are X and Y independent? (v) the correlation coefficient (vi) the conditional distribution of Y given $X = 2$.

OR

- Q.6(a) Do as directed:
- (i) Prove that two independent r.v's are uncorrelated but converse is not true.
- (ii) The joint p.d.f. of two r.v's X and Y is given by $f(x, y) = C(2x + y)$, $2 < x < 6$; $0 < y < 5$ and zero otherwise. Find (i) the constant C (ii) the marginal distribution of X (iii) $P(X > 3, Y > 2)$
- (b) Given $f(x, y) = k(x + 2y)$, $x, y = 1, 2$ and zero otherwise, is the joint p.m.f. of X and Y . Find (i) the constant k (ii) $P(X \geq 1, Y \leq 1)$ (iii) $P(X + Y \leq 2)$ (iv) $P(X < Y)$.

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