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SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester III) EXAMINATION -2022

US03CMTH52: Multivariate Calculus

Date: 16th November 2022, Wednesday

Time: 10:00 AM to 01:00 PM **Total Marks: 70 Marks**

Note: Figures to the right indicate full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. The value of $\int_1^\infty \frac{1}{x^2} dx$ is------

   a. 1 b. 0 c. -1 d. does not exist

2. If $\vec{V} = (3xyz)\bar{\imath} - (2x^2y)\bar{\jmath} + (2z)\bar{k}$ then $|div\vec{V}|$ at (1,1,1)----------

   a. 0 b. 3 c. 1 d. 2

3. The value of $B(m,n)$ is --------

   a. $\frac{n}{m+n}B(m,n)$ b. $\frac{n}{m+1}B(m,n)$ c. $\frac{m}{m+n}B(m,n)$ d. $\frac{m}{m+1}B(m,n)$

4. If we change the Cartesian variable (x,y) to the Cartesian variable (u,v) in double integral then $dxdy$= -----------

   a. $dxdy$ b. $Jdudv$ c. $|J|dudv$ d. $|J|dxdy$

5. In double integral total mass M of density 1 over region $0 \le x, y \le 2$ is ------------

   a. 1 b. 2 c. 0 d. 4

6. $\int_0^1 \int_0^2 dxdy =$ --------

   a. 1 b. 0 c. 3 d. 2

7. Area of plane region in polar form is given by $A =$ =--------

   a. $\frac{1}{2}\int_C r^2 d\theta$ b. $\int_C r^2 d\theta$ c. $\frac{1}{2}\int_C r d\theta$ d. $\frac{1}{2}\int_C [xdx - ydy]$

8. If $\bar{r} = u\bar{\imath} + v\bar{\jmath} + uv\bar{k}$ than $EG - F^2 =$ --------

   a. $1 + v^2$ b. uv c. $1 + u^2 + v^2$ d. $1 + u^2$

9. A function $f(x,y,z)$ is said to be harmonic if $\nabla^2 f =$ ----------

   a. 1 b. 2 c. -1 d. 0

10. If $\bar{n} = \bar{k}$ then $dA =$-----

    a. 0 b. $dxdz$ c. $dxdy$ d. $dydz$

Q:2 Answer in brief of the following questions. ***(Any Ten)*** **[20]**

1. Find the value of $\int_0^\infty x^2 e^{-x^4} dx$.
2. Let f be defined by $f(x,y,z) = x^2 siny + 1$. Find the directional derivative of the function f at $(0,0,0)$ in the direction of $(1,2,3)$.
3. Show that $curl(r^n\bar{r}) = 0$, where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $r = |\bar{r}|$.
4. Evaluate the line integral $\int_C 3(x^2+y^2)ds$ where C: along the circle $x^2+y^2 = 4$ from (2,0) to $(-2,0)$ (Anti clockwise).
5. Find the work done by force $4xy\bar{\imath} - 8y\bar{\jmath} + 2\bar{k}$ along the curve $(0,0,2)$ to $(3,6,2)$.
6. Find the area of the region bounded by $y = x^2$ and $y = 2x + 3$
7. Evaluate the line integral of $\int_{(0,1,2)}^{(2,\pi,0)}[(ydx + xdy)cosxy + dz]$ on any path.
8. Evaluate $\int_C \frac{\partial w}{\partial n} ds$ for the given w and C, $w = e^x cosy + x^3$, C: the boundary of the region $x^2 + y^2 \le 1$ in the first quadrant.
9. Identify the surface $\bar{r} = asinhucosv\bar{\imath} + bsinhusinv\bar{\jmath} + ccoshu\bar{k}$.
10. By using Divergence theorem evaluate the surface integral $\int_S\int [yzdydz + zxdzdx + xydxdy]$, where $S: x^2+y^2+z^2 = 1$.
11. Find the volume of the region R by using triple integration, R: In the first octant bounded by $x^2 + z^2 = 1$ and by the plane $y = 0, z = 0$ and $x = y$.
12. By using Stock's theorem evaluate $\int_C \bar{V}.\bar{t}ds$ where $\bar{V} = z\bar{\imath} + x\bar{\jmath}$, $S: 0 \le x, y \le 1, z = 1$.

Q:3 (a) State and prove the Duplication formula. **[05]**

(b) Prove: $\int_0^{\frac{\pi}{2}} \sqrt{sin\theta} \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sqrt{sin\theta}}} d\theta = \pi$ **[05]**

OR

(a) In usual notation prove that $curl(\bar{U}\times\bar{V}) = \bar{U}div\bar{V} - \bar{V}div\bar{U} + (\bar{V}.\bar{\nabla})\bar{U} - (\bar{U}.\bar{\nabla})\bar{V}$ **[05]**

(b) Let $f(r) = \frac{e^{\lambda r}}{r}$ be a scalar point function then prove that $(\nabla^2 - \lambda^2)f = 0$, where $r = \sqrt{x^2+y^2+z^2}$ **[05]**

Q:4 (a) Transform $\int_R\int (x-y)^2 sin^2(x+y)\, dxdy$ in uv –plane by taking $u = x - y$, $v = x + y$ where R: is the parallelogram with vertices $(\pi,0), (2\pi,\pi), (\pi,2\pi), (0,\pi)$. Hence evaluate it. **[05]**

(b) Let $f(x,y) = 1$ be the density of mass in the region $R: 0 \le y \le \sqrt{1-x^2}$, $0 \le x \le 1$, then find the center of gravity and moment of inertia I_x, I_y, I_0 **[05]**

OR

(a) Find the centroid of density 1 in the plane area bounded by $y = 6x - x^2$ and $y = x$ **[05]**

(b) Change the order of an integration $\int_0^{\frac{a}{2}} \int_{\frac{x^2}{a}}^{x-\frac{x^2}{a}} f(x,y)dydx$ **[05]**

Q:5 (a) State and prove Green's theorem. **[06]**

(b) Evaluate $\int_S \int f(x,y,z)dA$ where $f(x,y,z) = \tan^{-1}\left(\frac{y}{x}\right)$, $S: z = x^2 + y^2,\ 1 \le z \le 4,\ x, y \ge 0$ **[04]**

OR

(a) Verify both form (divergence and curl) of Green's theorem for the given $\bar{V}$ and C.
$\bar{V} = 7x\bar{\imath} - 3y\bar{\jmath}$ and C: the circle $x^2 + y^2 = 4$ **[06]**

(b) In usual notation prove that $\int_R \int \nabla^2 w dxdy = \int_C \frac{\partial w}{\partial n} ds$ **[04]**

Q:6 (a) State and prove Divergence theorem of Gauss. **[06]**

(b) By using the triple integration, find the total mass of distribution of density σ in a region R, where $\sigma = xy$ and R: the tetrahedral with vertices $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$ **[04]**

OR

(a) State and prove Stoke's theorem. **[07]**

(b) Evaluate: $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{a\sin\theta} \int_{z=0}^{\frac{(a^2-r^2)}{a}} r dr d\theta dz$ **[03]**

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