Seat No. : ____________ No. of pages: 03

[47]

SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester III) EXAMINATION -2022

US03CMTH51: Ordinary Differential Equations

Date: 15th November 2022, Tuesday

Time: 10:00 AM to 01:00 PM **Total Marks: 70 Marks**

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. Integral factor of differential equation of $cos^2x\frac{dy}{dx}+y=tanx$ is ----------

   a. e^{cosx} b. e^{tanx} c. $tanx$ d. $cosx$

2. Solution of the equation $y=px+\frac{a}{p}$ is given by ----------

   a. $y=cx$ b. $y=cx+\frac{a}{c}$ c. $y+cx=a$ d. $cy=x+a$

3. The solution of $p+2x=0$ is $y=$--------

   a. $c-\frac{x^2}{2}$ b. $c-x^2$ c. x^2-c d. $-2x$

4. $\frac{1}{D^2+9}cos3x=$-----------

   a. $\frac{x}{6}sin3x$ b. $-\frac{x}{6}sin3x$ c. $-\frac{x}{6}cos3x$ d. $\frac{x}{6}cos3x$

5. The complementary function of $(D^2-1)y=e^x+e^{-x}$ is-----------

   a. $C_1e^x-C_2e^{-x}$ b. $C_1e^x+C_2e^{-x}$ c. $(C_1+C_2x)e^x$ d. $(C_1+C_2x)e^{-x}$

6. The particular integral of $(D-7)^4=e^{7x}$ is--------

   a. $\frac{x^7}{24}e^{7x}$ b. $\frac{x^7}{24}e^{4x}$ c. $\frac{x^4}{24}e^{7x}$ d. $\frac{1}{24}e^{7x}$

7. If $L^{-1}\{f(s)\}=f(t)$, then $L^{-1}\{\bar{f}(s-a)\}=$--------

   a. $e^{at}f'(t)$ b. $e^{at}f(t)$ c. $f(t)$ d. None of these

8. $\int_0^\infty e^{-3t}sin4t\,dt=$---------

   a. $\frac{4}{25}$ b. $\frac{25}{4}$ c. $\frac{4}{13}$ d. $-\frac{4}{13}$

9. Orthogonal trajectories of $y^2=4a(x+a)$ is ---------

   a. $4c(x+c)$ b. $4(x+c)$ c. $4a(x+c)$ d. $4a(x+a)$

10. Kirchhoff's second law states that the sum of the voltage drop across inductor and resistor is -----

    a. different b. same c. inverse d. Not found

Q:2 Answer in brief of the following questions. *(Any Ten)* **[20]**

1. Solve: $(x^2 - 2xy - y^2)dx - (x+y)^2 dy = 0$
2. Solve: $p = tan\left(x - \frac{p}{1+p^2}\right)$
3. Eliminate the constant a and b by obtaining differential equation $y = e^x(acosx + bsinx)$
4. Find the particular integral (P.I.) of $(D^3 + 4D)y = 2x$
5. Find the complementary function (C.F.) of $(D^2 + 4)y = sin2x$
6. If y_1 and y_2 be two solutions of a linear differential equation $\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + - - + a_n y = 0$ and C_1 and C_2 are two arbitrary constants, then prove that $C_1 y_1 + C_2 y_2$ is also solution.
7. Find Laplace transform of $sin^3 2t$.
8. If $L[f(t)] = \bar{f}(s)$ then $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)ds$, provided integral exist.
9. State Linearity property and First shifting property.
10. A *12V* battery is connected to a simple series circuit in which the induction is $\frac{1}{2}H$ and resistance is 10Ω. Determine the current i if $i(0) = 0$
11. State Newton's law for cooling.
12. Find the curve through the point $(1,1)$ in the XY-plane having at each of its points having the slope $-\frac{y}{x}$.

Q:3 (a) Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ **[06]**

(b) Solve: $y(ydx - xdy) + 2x^2(ydx + xdy) = 0$ **[04]**

OR

(a) Solve: $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ **[05]**

(b) Define and solve Clairaut's equation, also solve $y^2 - 2pxy + p^2(x^2 - 1) = m^2$ **[05]**

Q:4 (a) Obtain the particular integral of $f(D)y = e^{mx}$, where m is a constant. **[06]**

(b) Solve $(D^2 - 5D + 6)y = 4e^x$ subject to the condition that $y(0) = y'(0) = 1$. Hence find $y(16)$ **[04]**

OR

(a) In usual notation prove that $\frac{1}{f(D)} xV = \left[x - \frac{1}{f(D)} f'(D)\right] \frac{1}{f(D)} V$, where V is a function of x **[05]**

(b) Solve: $x^3D^3 + 2x^2D^2 + 2y = 15(x - x^{-1})$ **[05]**

Q:5 (a) Evaluate: $L^{-1}\left\{\frac{s+2}{(s+3)(s+1)^3}\right\}$ [05]

(b) Evaluate: $L(t^2 sin^2 t)$ [05]

OR

(a) Solve: $\int_0^\infty \frac{cosat-cosbt}{t}dt$ [05]

(b) If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$ then $L^{-1}[\bar{f}(s).\bar{g}(s)] = \int_0^t f(u).g(t-u)du$

$= \int_0^t f(t-u).g(u)du$ [05]

Q:6 (a) It is found that 0.5 percentage of radium disappears in 12 years. (i) What percentage will disappear 1000 years? (ii) What is the half-life of radium? [05]

(b) 100 g. of a certain solvent is capable dissolving 50g. of a particular solute. Given that 25g. of the undissolve solute is contained in the solvent at time t=0 and that 10g. dissolved in 2 hours. Find the amount Q of the dissolve solute at any time t and t=6. [05]

OR

(a) Find the family orthogonal to the family $y = ce^{-x}$ of exponential curves. Determine the members of each family passing through (0,4) [05]

(b) Bacteria in a certain culture increase at a rate proportional to the number present. If the number doubles in one hour, how long does it take for number triple? [05]

———X———