Seat No. : .....

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C39J

SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester-III) Examination - 2022

US03CMTH22 (Multivariate Calculus)

Date: 16/11/2022, Wednesday

Time: 10:00 a.m. to 1:00 p.m. Total: 70 Marks

Q.1 Multiple choice questions. [10]

(1) The value of $\int_{-\infty}^{0} \frac{1}{\sqrt{3-x}}\, dx =$ .....

(a) 0 (b) ∞ (c) 3 (d) 1

(2) The value of $\int_{0}^{\infty} \frac{1}{x^2+4}\, dx$ .....

(a) 0 (b) 1 (c) $\pi/2$ (d) $\pi/4$

(3) The value of $\int_{-\infty}^{\infty} \frac{1}{4x^2+25}\, dx$ .....

(a) $\pi/10$ (b) $\pi/5$ (c) $\pi/2$ (d) π

(4) If we change Cartesian variable (x, y) to Cartesian variable (r, θ) in double integral then $dxdy =$ .....

(a) dudv (b) $rdrd\theta$ (c) $|J|\, dudv$ (d) $|J|\, dxdy$

(5) $\int_0^1 \int_0^1 dxdy =$ .....

(a) 1 (b) 0 (c) 3 (d) 2

(6) $\int_C [f dx + g dy + h dz]$ is independent of path iff $f dx + g dy + h dz$ is .....

(a) 0 (b) not exact (c) 1 (d) exact

(7) Area of plane region in Polar form is given by A = ...

(a) $\frac{1}{2}\int_C r^2 d\theta$ (b) $\int_C r^2 d\theta$ (c) $\frac{1}{2}\int_C r\, d\theta$ (d) $\frac{1}{2}\int_C [x\, dx - y\, dy]$

(8) $\int_0^1 \int_0^2 \int_0^3 dxdydz =$ .....

(a) 1 (b) 6 (c) 3 (d) 2

(9) $\int_0^1 \int_0^1 \int_0^1 x\, dxdydz =$ .....

(a) 1 (b) 0 (c) 2 (d) 1/2

(10) $\int_0^1 \int_0^1 \int_0^2 x\, dxdydz =$ .....

(a) 1 (b) 0 (c) 2 (d) 1/2

Q.2 Short Answer Question. (Attempt any TEN) [20]

(1) Find $\lfloor 15 \rfloor$.

(2) Prove that $B(7,5) = B(5,7)$.

(3) If $f(x,y,z) = 4(x+y) - 3z + 7y$, then find grad f at the point $(\sqrt{7}, \log 5, \frac{10}{700000})$.

(4) What is line integral dependent on path. Explain in short.

(5) Write the formula of work done by force.

(6) Evaluate $\int_C 2(x^2+y^2)ds$, where C: Over the path $y=x$ from $(0,0)$ to $(3,3)$ (counterclockwise direction)

(7) Evaluate: $\int_{(4,0,0)}^{(1,2,6)} [xdx+ydy+zdz]$

(8) Define: Line integral independent of path.

(9) Represent surface $\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$ in parametric form.

(10) State Stoke's theorem of Gauss.

(11) Evaluate $\int_0^{10}\int_0^{22}\int_0^{36} 5dxdydz$

(12) Evaluate $\int_0^4\int_0^3\int_0^2 xyz\,dxdydz$

Q.3(a) Find the directional derivative of surface x^2-xz+y at point $(1,-1,2)$ in the direction of vector $(2,3,9)$. [05]

(b) Prove that $\text{div}(\text{curl}\,\mathbf{V})=0$. [05]

OR

Q.3(c) Prove that $n\beta(m+1,n)=m\beta(m,n+1)$. [05]

(d) Prove that $\nabla^2(r^n\mathbf{r})=n(n+3)r^{n-2}\mathbf{r}$, where $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and $r=|\mathbf{r}|$. [05]

Q.4(a) Find area of the region bounded by bounded by $x^2=4y$ and $8y=x^2+16$ [05]

(b) Evaluate the double integral $\int_0^2\int_0^{\sqrt{4-x^2}} x^2\,dxdy$. [05]

OR

Q.4(c) Find volume of the region bounded by paraboloid $z=1-x^2-y^2$ and xy-plane. [05]

(d) Change the order of integration in $\int_0^7\int_1^y f(x,y)\,dxdy$. [05]

Q.5(a) Evaluate the surface integral of $\iint_S 3xy\,dA$ where, $S:\ z=x+y\ , 0\le x\ ,\ y\le 2$. [05]

(b) Find area of region $R: r=2(1-\sin\theta)$ [05]

OR

Q.5(c) Verify Green's theorem $\int_C [y^3\,dx+(x^3+3y^2x)\,dy]$, where C is the boundary of region bounded by $y=x^2$ and $y=x$ [10]

Q.6(a) State and prove divergence theorem. [05]

(b) Prove that Green's theorem is a special case of Stoke's theorem. [05]

OR

Q.6(c) Verify Stoke's theorem for $\bar{V}=3y\bar{i}-xz\bar{j}+yz^2\bar{k}$ and surface $S:\ 2z=x^2+y^2$ bounded by $z=2$. [10]