SEAT No.________

[46]

No. of Printed Pages: 3

Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-III : Examinations : 2022-23 [NC]

Subject : Mathematics | US03CMTH21 | Max. Marks : 70

Numerical Methods

Date: 15/11/2022, Tuesday | Timing: 10.00 am - 01.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] Mid-points of intervals are used for approximation of root of an equation while using the method of

[A] False position [B] Bisection

[C] Iteration [D] Aitkin's Δ^2-Process

[2] Newton-Raphson method is used for

[A] Interpolation

[B] Approximation of a root of an equation

[C] Approximation of derivative of a funcion

[D] None

[3] In usual notations, the formula $\xi = x_{i+1} - \dfrac{(\Delta x_i)^2}{\Delta^2 x_{i-1}}$ is used by the method of

[A] False position [B] Bisection

[C] Iteration [D] Aitken's Δ^2-Process

[4] $y_n - E^{-1}y_n =$

[A] Δy_{n+1} [B] ∇y_{n+1} [C] Δy_n [D] ∇y_n

[5] Which of the following is true?

[A] $\Delta y_5 = \nabla y_4$ [B] $\Delta y_5 = \nabla y_5$ [C] $\Delta y_4 = \nabla y_5$ [D] $\Delta y_6 = \nabla y_5$

[6] If $\nabla y_{10} = 10$ and $y_{10} = 25$ then $y_9 =$

[A] 15 [B] -15 [C] -5 [D] 5

[7] The divided differences are

[A] not dependent on their arguments

[B] symetrical in their arguments

[C] not symetrical in their arguments

[D] none

[8] In usual notations, we always have $[x_0, x_1]$ ---- $[x_1, x_0]$

[A] < [B] > [C] = [D] none

[9] For using Simpson's $\frac{1}{3}$ rule it is required that the number of sub-intervals be

[A] even [B] odd [C] a multiple of 3 [D] a multiple of 8

(P.T.O.)

[10] Which of the following method can be used to evaluate a numerical integral?

[A] Picard's Method [B] Euler's Method

[C] Runge-Kutta method [D] Romberg's Method

Q: 2. Answer any TEN of the following. **20**

[1] Find an interval containing an initial approximation of $x^3 - 4x + 1 = 0$

[2] Find an interval containing an initial approximation of $5\sin x + 3 = 0$

[3] Find an interval containing an initial approximation of $\tan x = 1$

[4] Prove that $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

[5] Prove that $\Delta = E\nabla$

[6] If $E^{10}y_1 = 20$ then find $E^5y_6 + E^6y_5$

[7] Construct divided difference table for the data

x	2	3	4	5
y	10	15	18	20

[8] If $y_1 = 4,\ y_3 = 12,\ y_4 = 19$ and $y_x = 7$ find x. Write the formula you use and also give it's name

[9] For the given data

x	5	7	8
y	2	5	6

find $y(6)$

[10] Using Trapezoidal rule find $\int_0^5 \frac{1}{x+1}dx$, with subintervals of length 1 unit.

[11] Given that $\frac{dy}{dx} = x^3 + y,\ y(0) = 1$, determine $y(0.02)$ using Euler's method, , taking $h = 0.01$

[12] Using Trapezoidal rule find $\int_0^3 e^x dx$, with 3 subintervals of equal lengths.

Q: 3 [A] Using Bisection method find a real root of the equation $x^3 - 10x + 3 = 0$ correct upto four decimal palaces **5**

[B] Find a real root of $x^3 - 4x - 9 = 0$ by method of False Position correct upto three decimal places **5**

OR

Q: 3 [A] Using Bisection method find a real root of the equation $2x\log_{10}(x+5) - 6 = 0$ correct upto three decimal palaces 5

[B] State and prove the condition on $\phi(x)$ in Iteration method for convergence of a sequence of approximations. 5

Q: 4 [A] Derive Newton's Backward Difference interpolation formula for equally spaced values of argument 5

[B] Derive Gauss's Forward interpolation formula for equally spaced values of argument 5

OR

Q: 4 [A] Use Stirling's formula to find u_{32}, given that

$$u_{20} = 14.035,\ u_{25} = 13.674,\ u_{30} = 13.257,\ u_{35} = 12.734,\ u_{40} = 12.089 \text{ and } u_{45} = 11.309$$

5

[B] Let $y = g(x)$ be a function such that

$$g(20) = 2854,\ g(24) = 3162,\ g(28) = 3544,\ g(32) = 3992$$

Use Everett's formula to obtain $g(25)$. 5

Q: 5 [A] Certain corresponding values of x and $\log_{10} x$ are

$$(300, 2.4771),\ (304, 2.4829),\ (305, 2.4843),\ \text{and } (307, 2.4871)$$

Find $\log_{10}(301)$. 5

[B] Show that the divided differences are symetrical in their arguments 5

OR

Q: 5 [A] Discuss the method of succesive apprximation for inverse interpolation. 5

[B] Given the set of tabulated points (x,y) which are $(1, -3)$, $(3, 9)$, $(4, 30)$ and $(6, 132)$ obtain the value of y when $x = 2$ using Newton's divided difference formula 5

Q: 6 [A] Derive the general formula for Trapezoidal rule 5

[B] Use Picard's method to approximate y when $x = 0.25$, given that $y(0) = 0$ and $\dfrac{dy}{dx} = \dfrac{x^2}{y^2+1}$ correct upto three decimal places 5

OR

Q: 6 [A] Using Euler's method solve $y' = -y$, taking five subintervals and $h = 0.01$ with initial condition $y(0) = 1$. 5

[B] Given that $\dfrac{dy}{dx} = y - x$, $y(0) = 2$, determine $y(0.1)$ and $y(0.2)$ using Runge-Kutta method, correct upto four decimal places 5

—x—