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SARDAR PATEL UNIVERSITY

B.Sc. (SEMESTER - III) EXAMINATION-2022 (On demand)

20/06/2022 Monday

12:00 p.m. to 2:00 p.m.

US03SMTH21 (Number Theory I)

Maximum Marks: 50

Q.1 Multiple choice question. [08]

(1) $(a, b) \geq$ ....
(a) a (b) 2 (c) 1 (d) 0

(2) If $a = bc$; $a, b, c \in \mathbb{Z}$ then
(a) a/c (b) a/b (c) b/c (d) c/a

(3) If n is odd integer then $3^n + 1$ is divisible by ....
(a) 5 (b) 3 (c) 4 (d) 6

(4) If a/k , b/k , $k > 0$ then $\left(\frac{k}{a}, \frac{k}{b}\right) =$ ....
(a) 1 (b) $k\,(a\,,\,b)$ (c) $\frac{k}{(a\,,\,b)}$ (d) $\frac{k}{[a\,,\,b]}$

(5) Any prime factor of M_p is .... p.
(a) $<$ (b) $=$ (c) $>$ (d) $\leq$

(6) $T(11) =$....................
(a) 3 (b) 12 (c) 2 (d) 11

(7) $\mu(12) =$ ....
(a) 1 (b) 0 (c) -1 (d) 3

(8) If a is prime then $P(a) =$ ....
(a) a (b) $a - 1$ (c) 1 (d) $a + 1$

Q.2 Do as directed. [06]

(1) The GCD of (-2,-6) = ...

(2) If b/a then $(a\,,\,b) = |b|$ $\forall$ $a\,,\,b \in \mathbb{Z}$. (True or False?)

(3) The LCM of [12,30] = ...

(4) 111 is a prime number.(True or False?)

(5) $S(11) = 12$.(True or False?)

(P.T.O.)

(6) $\mu(10) =$ ....

Q.3 Answer the following questions in short. (Attempt any 6) [12]

(1) Prove that $(a-s)/(ab+st) \Rightarrow (a-s)/(at+sb)$.

(2) Prove that $c/a, c/b \Rightarrow c/(ma+nb) \quad \forall \ m,n \in Z$.

(3) Find $P(50)$.

(4) Prove that $[a,b,c] = [[a,b],c]$.

(5) Define Prime number with example.

(6) Find highest power of 2 in 40! i.e 2(40!)

(7) Find number of multiple of 7 among the integers from 100 to 400.

(8) Find [140,145,155].

Q.4 Long Questions (Attempt any 4) [24]

(1) State and prove Fundamental theorem of divisibility.

(2) Prove that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$.

(3) State and prove the relation between G.C.D. and L.C.M. of two numbers.

(4) If P_n is n^{th} prime number then prove that $P_n < 2^{2^n}$, $\forall n \in \mathbb{N}$.

(5) State and prove unique factorization theorem for positive integers.

(6) If a is a square number then prove that $S(a)$ is odd integer.

(7) In usual notation prove that $u_{m+n} = u_{m-1}u_n + u_m u_{m+1}$, $\forall \quad m,n \in \mathbb{N}$

(8) Let x be any positive real number and n be any positive integer then prove that among the integers from 1 to x the number of multipliers of n is $\left[\frac{x}{n}\right]$.

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