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[61] **SARDAR PATEL UNIVERSITY(B.Sc. Sem.3 On Demand Examination (NC))**

MATHEMATICS - US03EMTH05 – Calculus and Algebra-1

14th June 2022, Tuesday

No of printed pages :02

Time Duration: 02 Hours - 12.00 TO 02.00 PM **Maximum Marks: 70**

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. **[10]**

1. $\lim_{x\to 0}(cotx)^{sin2x}$ is of the form _____
   (a) $\frac{\infty}{\infty}$ (b) $\frac{0}{0}$ (c) 1^{∞} (d) ∞^0
2. $\lim_{x\to 0}(\frac{1}{2x^2}-\frac{cot^2x}{2})$ is of the form _____
   (a) 0^0 (b)$\infty-\infty$ (c) $0-0$ (d) ∞^{∞}
3. If $u=x^3-3xy^2$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$ _____
   (a) 0 (b) $3u$ (c) $2u$ (d) nu
4. $u=\frac{x^2y^2}{x+y}$ is a homogeneous function of degree _____
   (a) 1 (b) 2 (c) 3 (d) 4
5. A matrix of order (3×3) has _____elements
   (a) 3 (b) 4 (c) 6 (d) 9
6. A column matrix is of order _____.
   (a) $m\times n$ (b) $m\times 1$ (c) $1\times n$ (d) 3×1
7. In an identity matrix, all the diagonal elements are_____.
   (a) 1 (b) $\emptyset$ (c) i (d) 0
8. A is a square matrix then A + A^T is _____.
   (a)zero matrix (b) unit matrix (c) symmetric (d) skew- symmetric
9. $(A^T)^T=$ _____.
   (a) A^{-1} (b) I (c) A^T (d) A
10. For matrix multiplicationAB_____BA
    (a) = (b) $\neq$ (c) $\subset$ (d) $\supset$

Q.2 Answer the given statement is TRUE or FALSE **[08]**

1. $\lim_{x\to\frac{\pi}{2}}\frac{\log(\cos x)}{\log(\cos 3x)}$ is an indeterminate form of $\frac{\infty}{\infty}$.
2. 0×0 is an indeterminate form.
3. If $z=x^3$ then $\frac{\partial^2 z}{\partial y^2}=6x$
4. If $f(x,y)=x^2y-xy^2$ then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=3f(x,y)$
5. In diagonal matrix all the non- diagonal elements are zero.
6. Triangular matrices are always square matrices.
7. Zero matrix is always a square matrix.
8. $|I|=1$.

(P.T.O.)

Q-3 Answer ANY TEN of the following. [20]

1. Evaluate $\lim_{x\to 3} \frac{x^2-9}{x^3-27}$
2. Evaluate $\lim_{x\to 0} \frac{\log \sin x}{\cot x}$
3. Evaluate $\lim_{x\to 2} \frac{\sin (x^2-4)}{(x-2)}$
4. If $f(x,y) = xtan\, y + ytan\, x$ then find f_{yx}
5. Check whether $z = \frac{xy}{x+y}$ is homogeneous or not. If yes, find its degree.
6. If $u = x^3 - 3xy^2$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
7. If A is Hermitian then prove that iA is skew-Hermitian
8. If A and B are symmetric matrices then prove that $A - B$ is symmetric.
9. If $A = \begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$ then find BA
10. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then prove that $A^2 - 4A + 5I = 0$
11. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then prove that $AA^T = I$
12. Find determinant of matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 2 & 6 & 4 \end{bmatrix}$

Q-4 Answer ANY FOUR of the following. [32]

1. Evaluate $\lim_{x\to a}(a-x)\tan(\frac{5\pi x}{2a})$
2. Evaluate $\lim_{x\to 0} \frac{e^x+\log(1-x)-1}{\tan x - x}$
3. Verify Euler's theorem for $z = x^2y - xy^2$
4. If $H = f(y-z, z-x, x-y)$ then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$
5. If A and B are symmetric matrices of the same order then prove that AB is symmetric iff A and B are commutative.
6. Verify $AB = BA$ for $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$
7. State and prove Reversal law for the transpose of a product of two matrices.
8. Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

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