[63] Seat No.______ S

No of printed page : 2

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - III (NC)) EXAMINATION - 2022
Saturday, 18^{th} June 2022 MATHEMATICS: US03EMTH01
(CALCULUS)

Time : 12:00 Noon to 02:00 p.m. **Maximum Marks : 70**

Que.1 Fill in the blanks. 10

(1) $\int_1^\infty x dx$ is ....
(a) Proper Integral (b) Improper integral of 1st kind
(c) Improper integral of 2nd kind (d) none

(2) the integral $\int_a^\infty \frac{dx}{x^\mu}$ $(a > 0)$ is convergent if and only if ....
(a)$\mu > 1$ (b)$\mu < 1$ (c)$\mu = 1$ (d) None

(3) Gamma function defined as $\Gamma n=$ ....
(a)$\int_0^\infty x^{n-1}e^{-x}dx$ (b)$\int_0^1 x^{n-1}e^{-x}dx$ (c)$\int_0^\infty x^{n-1}e^{x}dx$ (d) $\int_0^1 x^{n+1}e^{-x}dx$

(4) The improper integral $\phi(X)=\int_a^\infty f(X)dx$ is convergent iff ....
(a) $\phi(X) \geq k$ (b) $\phi(X) \leq k$ (c) $\phi(X) = k$ (d) None

(5) $\int_0^1 \frac{1}{x}dx$ is unbounded at x= ....
(a) 1 (b) -1 (c) 2 (d) 0

(6) $\int_0^{\pi/2} \sin^p \cos^q d\theta$ is ....
(a) $\frac{1}{2}\beta\left(\frac{p}{2},\frac{q}{2}\right)$ (b) $\frac{1}{2}\beta\left(\frac{p-1}{2},\frac{q-1}{2}\right)$ (c) $\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right)$ (d) $\frac{1}{2}\beta\left(\frac{p-1}{2},\frac{q+1}{2}\right)$

(7) $\nabla^2 = \sum \frac{\partial^2}{\partial x^2}$ is called ....
(a) Laplacian operator (b) Gradient of scalar field
(c) Vector differential operator (d) Divergence vector field

(8) $\overline{\nabla} \times \bar{v} =$ ....
(a) $\Sigma\bar{i} \times \frac{\partial \bar{v}}{\partial x}$ (b) $\Sigma\bar{i}\frac{\partial \bar{v}}{\partial x}$ (c) $\Sigma\bar{i}x$ (d) $\Sigma\frac{\partial \bar{v}}{\partial x}$

(9) If f is scalar function then $\overline{\nabla} \times (\overline{\nabla} f) =$ ....
(a) 1 (b) -1 (c) $\bar{0}$ (d) $\nabla^2 f$

(10) The periodic function $f(x)$ is defined as ....
(a) $f(x - T) = f(x)$; T is some positive number
(b) $f(x \cdot T) = f(x)$; T is some positive number
(c) $f(x + T) = f(x)$; T is some positive number
(d) None

Que.2 Write TRUE or FALSE. [8]

(1) $\int_0^{\pi/2} \sin x dx$ is Improper integral of 1st kind.

(2) If $n = 4$ then $\Gamma n = 6$

(3) $\overline{\nabla} f = \sum \bar{i}\frac{\partial f}{\partial x}$

(4) $\overline{\nabla} \cdot \overline{v} = \Sigma \frac{\partial \bar{v}}{\partial x}$

(5) Beta function defined as $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

(6) Primitive period of $\sin mx$ is $\frac{2\pi}{m}$

(7) Graph of even function has symmetry with respect to X - axis.

(8) Product of two odd functions is Constant.

Que.3 Attempt the following (Any TEN) [20

(1) Evaluate $\int_1^\infty \frac{dx}{1+x^2}$.

(2) Evaluate $\int_1^\infty \frac{dx}{\sqrt{x}}$.

(3) Prove that $\beta(m,n) = \beta(n,m)$.

(4) Prove that $\left\lceil \frac{1}{2} \right. = \sqrt{\pi}$

(5) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p,q)}{p+q}$

(6) Find gradient of $f(x,y) = \frac{x}{x^2+y^2}$ at (2 , 3) .

(7) Find gradient of $f(x,y,z) = (x^2+y^2+z^2)$ at (1 , 2 , 3) .

(8) Evaluate $\int_0^\infty e^{-x^2}dx$.

(9) Prove that $\overline{\nabla}(f \pm g) = \overline{\nabla} f \pm \overline{\nabla} g$.

(10) Prove that $\overline{\nabla}(fg) = f\overline{\nabla} g + g\overline{\nabla} f$

(11) Find the primitive period of the following function $f(x) = \sin 2x$.

(12) Find the smallest period of the following function $f(x) = \cos \pi x$.

Que.4 Attempt the following (Any FOUR) [32

(1) Prove that the integral $\int_a^b \frac{dx}{(x-a)^\mu}$ is convergent if and only if $\mu < 1$.

(2) Examine convergence of $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$

(3) Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx$.

(4) Evaluate $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}$

(5) Find directional derivative of $f(x,y,z) = 2x^2+3y^2+z^2$ at point (2,1,3) in the direction of $\bar{a} = \bar{i} - 2\bar{k}$.

(6) Prove that $\overline{\nabla}^2(fg) = f\overline{\nabla}^2 g + g\overline{\nabla}^2 f + 2\overline{\nabla} f \cdot \overline{\nabla} g$.

(7) Find the Fourier coefficient of the periodic function $f(x) = x^2/4$, $-\pi < x < \pi$.

(8) Find the Fourier coefficient of the periodic function : $f(x) = \begin{cases} 1 & \text{if } -\pi/2 < x \leq \pi/2 \\ -1 & \text{if } \pi/2 < x < 3\pi/2. \end{cases}$

—x—