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SEAT No ________ No. of Printed Pages: 3

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SARDAR PATEL UNIVERSITY
S.Y.B.Sc. (Semester-III) Examination
16th June, 2022
Thursday

US03CSTA22

Subject:-Elements of Probability Theory

16/06/22

Time:- 12:00 P.M. to 02:00P.M. **Marks:- 70**

Note:- Simple/Scientific calculators are allowed. Statistical Table is allowed.

Q.1. Multiple Choice Questions: - [10]

1 If A and B are mutually disjoint events then P(AUB) = __________.
(a) P(A)*P(B) (b) P(A)+P(B) (c) 0 (d) Ø

2 Four coins are tossed, the number of sample points in a sample space is ______.
(a) 16 (b) 12 (c) 24 (d) none

3 The interval (Q_2 ,Q_3) includes ________ % of points.
(a) 25 (b) 50 (c) 75 (d) none

4 If f(x) is the p.d.f of the continuous random variable X and if $P(X \le M) = P(X \ge M)$ then M = _______.
(a) mean (b) median (c) mode (d) none

5 If $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-3x}, & x \ge 0 \end{cases}$, is the c.d.f. of of the random variable X then f(x) = ___________.
(a) $-3e^{-3x}$ (b) $\frac{-1}{3}e^{-3x}$ (c) $3e^{-\frac{1}{3}x}$ (d) none

6 If $M_X(t)$ is the m.g.f. of a random variable X and Y = aX + b then $M_Y(t)$ = ____.
(a) $e^{bt}M_X(at)$ (b) $e^{at}M_X(bt)$ (c) $M_X(at+b)$ (d) none

7 If Px(t) is the p.g.f. of a discrete random variable X then P(X = x) = _____.
(a) $\frac{dP(t)}{dx}|_{t=1}$ (b) $\frac{dP(t)}{dt}|_{t=0}$ (c) $\frac{dP(t)}{dt}|_{t=-1}$ (d) none

8 If X and Y are two independent random variables with E(X) = 9 and E(X.Y) = 36 then E(Y) = _______.
(a) 6 (b) 4 (c) 2 (d) none

9 If two random variables X and Y are independent then E(X.Y) = _______.
(a) E(X) + E(Y) (b) E(X).E(Y) (c)) E(X)/E(Y) (d) none

10 If f(x,y) = 4xy , 0 < x < 1 ; 0 < y < 1,
= 0 , elsewhere is the joint p.d.f. of X and Y then
P(X < 0.50 , Y < 0.50) = _________.
(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c)) $\frac{1}{64}$ (d) none

Q.2. Fill in the blanks: - [08]

1 Two balls are drawn at random, WR from a box containing 6 red and 4 green balls. The probability that first ball is green and second is red _______.

2 The value of k=_______ for

x	1	2	3	4	5	6
F(x)	k	4k	9k	16k	25k	36k

3 If X and Y are two independent variables then V(X-Y)=_________.

4 P(X=xi, Y=yj) = P(X=xi) X P(Y=yj) for every i and j , then X and Y are __________.

(1) (P.T.O.)

State whether the statement is True or False.

5 The probability of throwing coin is 1/6.

6 The probability generating function is given by $P_x(t) = \sum_x e^{xt}\, p(x)$.

7 The m.g.f. of sum of two random variables is the product of their mgf's.

8 For bi variate prob. distribution of X and Y $\sum_x \sum_y f(x,y) \neq 1$.

Q.3. Short Questions: - (Attempt any Ten) [20]

1 The probability of simultaneously occurrence of at least one of two events A and B is p. If the probability that exactly one of A,B occur is q, then prove that $P(A') + P(B') = 2 - 2p + q$.

2 Two balls are drawn at random from the box containing 3 white, 2 black and 4 green balls. Find the probability that both the balls drawn are of different colors.

3 If P(A) = P(B) = ½ and P(AUB) = 2/3

Find (i)P(exactly one of A and B occurs) (ii)P(none of A and B occurs).

4 The table below shows the prob. of X

x	1	2	3	4	5
p(x)	2k	3k	4k	5k	6k

For what value of k is the prob. is p.m.f or probability function.

Find $P(3 \le X \le 5)$

5 Consider the experiment of tossing of three fair coins. Let variable X denote the number heads. Find the probability mass function and cumulative distribution function.

6 If F(x) = 0 , x<1

$= \frac{x-1}{12}$, $1 \le x < 13$

= 1, $x \ge 13$, is the c.d.f. of a random variable X then find the pdf of X and Q_1, Median and P_{75}.

7 Let X be a r.v. with the following prob. distribution:

x	-3	6	9
p(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E(x) and $E(x^2)$, Evaluate $E(2X+1)^2$.

8 If $f(x) = \frac{x}{21}$, x = 1,2,3,.....,6.

= 0, otherwise, is the pmf of a discrete r.v. X then find E(X),V(X),E(2x+3).

9 If X and Y are two r.v.'s then obtain cov(ax,by).

10 Is $f(x,y) = \frac{(2x+3y)}{120}$, x=1,2,3; y=1,2

= o, otherwise, the joint p.m.f. of X and Y?

11 If f(x,y) = k (3 x + 2 y) , x = 1,2,3,4; y=1,2,3

= 0 , otherwise is the joint p.m.f. of X and Y.

Find (i) k (ii) the marginal distribution of X and Y.

12 The joint p.d.f. of a two dimensional random variables (X,Y) is given by

.f(x,y) = $\frac{1}{8}(6-x-y)$; $0 \le x < 2$, $2 \le y < 4$

= 0, otherwise

Find the $P(X<1 \cap Y<3)$ and $P(X+Y<3)$.

Q.4. Long Questions: - (Attempt any four) [32]

1 (a)Two dice are thrown together. Find the probability of getting

(2)

(i)an even number on first die or a total of 8 in single throw
(ii)neither divisible by 3 nor by 4.
(b)Two students X and Y appeared in examination. The prob. that X will qualify the examination is 0.05 and that Y qualify the examination is 0.10. The prob. that both will qualify the examination is 0.02. Find the prob. that (i) both X and Y will not qualify the exam (ii) at least one of them will qualify the exam (iii) only one of them will qualify the exam (iv) at least one of them will not qualify.

2 (a)State and Prove addition law of probability for two events.
(b)(i) For any two events A and B, prove that If $A \subset B$, then prove that $P(A) \leq P(B)$
(ii)$P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

3 If f(x) = kx, 0 < x < 2
= k(5-x) , 2<x< 5
= 0, otherwise is the pdf of x then find
(i) k (ii) $P(1 < x \leq 2)$ (iii) $P(1 \leq x \leq 3)$ (iv)$P(x \geq 4)$.(v) cdf of f(x)

4 A discrete r.v. has the pmf

x	1	2	3	4	5	6	7
p(x)	k	2k	2k	3k	$2k^2 + k$	$3k^2$	$5k^2$

Determine k (ii) $P(X \leq 4)$ (iii) $P(2 < X < 7)$ (iv) cdf of f(x).(v)Find the min. value of a such that $P(X \leq a) < 0.5$.(vi) $P(2 \leq X \leq 5 / X \leq 4)$

5 If f(x) = 6 x (1-x), 0 < x < 1
= 0, otherwise
Calculate μ_r' and hence find Mean and variance, $\beta_1$and β_2.Also find Mode.

6 Two numbers are selected at random, without replacement from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the mean , variance, μ_3' and μ_4', $\beta_1$and β_2.

7 Suppose that two dimensional continuous random variable (X,Y) has joint p.d.f. given by
.$f(x,y) = 6x^2 y$, 0<x<1 , 0<y <1
= 0 , otherwise
(i)Verify given joint function is p.d.f.
(ii) Find $P(0<X<\frac{3}{4}, \frac{1}{3}<Y<2)$,(iii) P(X+Y <1) (iv) P(X> Y) (v) P(X<1| Y<2) .

8 State the condition for independent for two variables X and Y.
If $f(x,y) = \frac{xy}{36}$, x = 1,2,3; y=1,2,3
= 0 , otherwise is the joint pmf of X and Y.
(i)Show that X and Y are independent.(ii) P(X <2,Y<3) (iii) P(X<3,Y<2)(iv)P(X<2/Y<2).
(v) Marginal p.d.f. of X and Y variables.

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