**SARDAR PATEL UNIVERSITY**

B.Sc Semester – III CBCS (NC) Examination

Thursday, 16th June, 2022

Time : 12 to 02 PM          Statistics          M.Marks : 70

**US03CSTA02 (Elements of Probability Theory)**

1    Let $X$ and $Y$ have the joint prob. mass function

$f(x,y) = \frac{x+y}{21}, x = 1,2,3 ; y = 1,2$ *and zero otherwise*, then $P(X = 2/Y = 2)$ equals

(a) 1/3          (b) 2/3          (c) 1/2          (d) 1/4

2    If $K$ is a constant then $V(K)$ will be

(a) 0          (b) $K$          (c) $\infty$          (d) cannot be determined

3    $A$ and $B$ are two events, the probability that exactly one of them occurs is given by

(a) $P(A \cup B) - P(A \cap B)$          (b) $P(A \cap B') + P(A' \cap B)$

(c) $P(A) + P(B) - 2P(A \cap B)$          (d) All of the above

4    The pdf of a r.v. $X$ is then $P(X \leq 2) =$ _____

$$f(x) = \begin{cases} \frac{x}{3}, 0 \leq x < 1 \\ \frac{5}{27}(4 - x), 1 \leq x < 4 \\ 0, otherwise \end{cases}$$

(a) 1/3          (b) 5/27          (c) 17/27          (d) 20/27

5    If $M_X(t) = (1 - 3t)^{-1}$ then $E(2 + 3X) =$ _____

(a) 4          (b) 9          (c) 8          (d) 11

6    If $X$ and $Y$ are two independent variates then

(a) $Cov(X,Y) = 0$          (b) $f_{XY}(x,y) = f_X(x)f_Y(y)$

(c) $f_{Y/X}(y/x) = f(y)$          (d) All of the above

7    Two dice are rolled. What is the probability of getting two numbers whose product is prime number?

(a) 1/6          (b) 1/12          (c) 3/8          (d) 5/16

8    If $f(x) = x/15, x = 1,2,3,4,5$ *and* zero otherwise, then $P(X > 3) =$ _____

(a) 2/15          (b) 3/15          (c) 5/15          (d) 9/15

9    If $P(A) = 1/4, P(B) = 2/5$ and $P(A \cup B) = 1/2$ then $P(A^c \cup B^c) =$ _____

(a) 0.85          (b) 0.58          (c) 0.80          (d) 0.15

10    The joint pdf of $X$ and $Y$ is $f(x,y) = (xy)/4, 0 < x < 2, 0 < y < 2$ *and zero otherwise*, then the value of $P(0 < X < 1 \cap 1 < Y < 2) =$ _____

(a) 1/16          (b) 2/16          (c) 3/16          (d) 5/16

Q.2    <u>Fill in the blanks</u>          (4 × 1)

1    Two events $A$ and $B$ are mutually exclusive if $P(A \cap B) =$ _____ and are independent if $P(A \cap B) =$ _____

2    Let $X$ and $Y$ be two random variables with joint pdf

$f(x,y) = c(2x + y), 0 < x < 1, 0 < y < 2$ *and zero otherwise*, then $c =$ _____

3    The pdf of a r.v $X$ is $f(x) = \begin{cases} 1 + x, -1 < x < 0 \\ 1 - x, 0 < x < 1 \end{cases}$

①          (P.T.O)

then $P(0.2 < X < 0.4) =$ _____

4     Let a r.v. $X$ denote the waiting time of a passenger at a railway station then $X$ is a _____ type of r.v.

<u>True – False</u>                                                $(4 \times 1)$

5     The probability of occurrence of at least one of two events is the sum of the probability of each of the two events.

6     If $A$ and $B$ are two events then $P(A \cap B^c) = P(B) - P(A \cap B)$

7     If $X$ and $Y$ are two independent r.v's then they are uncorrelated.

8     The m.g.f of sum of two random variables is the product of their m.g.f's.

**Q.3**     **Short Type Questions (Attempt Any Ten)**                        $(10 \times 2)$

1     An urn contains 7 white and 3 red balls. Two balls are drawn at random, without replacement from this urn, find the probability distribution of $X$ where $X =$ no. of red balls drawn.

2     Prove that the complementary events of two independent events are also independent.

3     If $V(X + Y) = 3, V(X - Y) = 1, E(X) = 1, E(Y) = 2$ then find $E(XY)$.

4     Define Moment Generating Function (m.g.f). Write down its properties.

5     The p.g.f. of $X$ is $P(t) = (3 - 2t)^{-1}$ then find $(i)$ the probability distribution of $X$
      $(ii)$ $P(X \leq 2)$

6     If $f(x) = 3x^2, 0 < x < 1$ and zero otherwise, is the pdf of $X$. Find Inter Quartile Range.

7     The joint p.d.f. of $X$ and $Y$ is

$$f(x,y) = \frac{2}{3}(x + 2y), 0 < x < 1, 0 < y < 1 \text{ and zero otherwise.}$$

      Determine $P(X < 1/2, Y < 1/2)$

8     Let $X$ be a r.v having pdf $f(x) = kx^2, |x| < 1$ and zero otherwise. Determine $k$.

9     A fair six – sided die is rolled twice. What is the probability of getting 2 on the first roll and not getting 4 on the second roll?

10     Determine $k$, if the p.m.f. of a r.v. $X$ is

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|------|------|------|------|------|------|------|
| $P(x)$ | $k^2$ | $k/4$ | $5k/2$ | $k/4$ | $2k^2$ | $k^2$ |

11     Examine whether $f(x,y) = 6x^2 y, 0 < (x,y) < 1$ and zero otherwise, is joint pdf of $X$ and $Y$ or not.

12     If $M_x(t) = e^{2(e^t - 1)}$ is the m.g.f of a random variable $X$, find $V(2X + 3)$.

**Q.4**     **Long Answer Questions (Attempt Any Four)**              $(4 \times 8)$

**(a)**     $(i)$ Let $X$ be a r.v. with the following probability distribution:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|------|------|------|------|------|------|------|------|
| $P(x)$ | 1/20 | P1 | 1/5 | P2 | P3 | 1/10 | 1/10 |

      If $E(X) = 3.1, E(X^2) = 12.1$. find P1, P2 and P3.

      $(ii)$ Let $A$ and $B$ be two independent events. The prob. of simultaneous occurrence of these events is $1/8$ and the probability of none of these occurs is $3/8$. Find $P(A)$ and $P(B)$.

**(b)**     The probability distribution of a r.v. $X$ is given below:

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| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|---|---|---|---|---|---|---|---|
| $P(x)$ | $k$ | $k$ | $3k$ | $2k$ | $k^2 + k$ | $2k^2$ | $4k^2 + k$ | $3k^2$ |

Find $(i)$ $k$ $(ii)$ the c.d.f. of $X$ $(iii)$ the minimum value of $k$ so that $P(X \leq k) = 1/2$
$(iv)$ $P(2 \leq X < 7)$ $(v)$ $P(X > 3)$.

(c) State and prove addition law of probability for two events.
Using this, prove that : $(i)$ $P(A^c) = 1 - P(A)$ $(ii)$ $P(A \cap B) \geq P(A) + P(B) - 1$

(d) $(i)$ The coefficient of correlation between $X$ and $Y$ is $1/3$, $V(X) = K, V(Y) = 4K$ and
$V(Z) = 114$, Where $Z = 3X - 4Y$. Find the constant $K$.
$(ii)$ The joint pdf of $X$ and $Y$ is
$$f(x,y) = 4xye^{-(x^2+y^2)}, x > 0, y > 0 \ and \ zero \ otherwise.$$
$(i)$ Test whether $X$ and $Y$ are independent or not $(ii)$ find the conditional distribution of $X$
given $Y$.

(e) Verify whether the following function is p.d.f. or not?
$$f(x) = \begin{cases} \dfrac{x}{3}, & 0 \leq x < 1 \\ \dfrac{5}{27}(4 - x), & 1 \leq x < 4 \\ 0, & otherwise \end{cases}$$
If yes, then find $(i)$ the distribution function of $X$ $(ii)$ $P(0.5 < X < 2)$

(f) Let $X$ and $Y$ have the joint density function
$$f(x,y) = x + y, 0 < (x,y) < 1 \ and \ zero \ otherwise$$
$(i)$ Find the correlation coefficient between $X$ and $Y$ $(ii)$ Are $X$ and $Y$ independent?
$(iii)$ Calculate $P(X > 1/3)$

(g) Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the mean
and variance of the no. of kings drawn.

(h) Following are the compositions of two basket flowers:

| Basket - I | Basket - II |
|-----------|-------------|
| Pink - 5 | Pink - 4 |
| White – 3 | White – 5 |
| Yellow - 2 | Yellow – 4 |

One basket is chosen at random and two flowers drawn from it they happen to be white
and yellow. What is the probability that they come from basket – I, II?

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