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B.Sc. (III-Semester) EXAMINATION 2022

Tuesday, 14th June, 2022

12:00pm 2:00pm

US03CMTH 22-Mathematics

MULTIVARIATE CALCULUS

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: [10]

1. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}} =$ -----------

   a. 0 b. 1 c. $\frac{\pi}{2}$ d. ∞

2. The value of m for which the vector field $\vec{F} = (2x+y)\bar{\imath} + (3x-2z)\bar{\jmath} + (x+mz)\bar{k}$ is solenoid

   a. 0 b. 2 c. -2 d. 1

3. $\beta(m+1,n) + \beta(m,n+1) =$ ----------

   a. $\beta(m,n)$ b. $\beta(m+1,n)$ c. $\beta(m,n+1)$ d. $\beta(m+1,n+1)$

4. In double integration volume of $f(x,y)$ over region R is given by V=------

   a. $\int \int_R dxdy$ b. $\int \int_C dxdy$ c. $\int \int_R f(x,y)\,dxdy$ d. $\int \int_R dxdydz$

5. Work done by force $\bar{P}$ over the curve C is given by W=

   a. $\int_C \bar{P}.d\bar{r}$ b. $\int_C \bar{P}\, d\bar{r}$ c. $\int_C \bar{P}\, dxdy$ d. $\int_C \bar{P}\, dxdydz$

6. If $x = rcos\theta, y = rsin\theta$ then Jacobin J= ------

   a. 1 b. r^2 c. r d. 2

7. The surface $\bar{r} = aucosv\vec{\imath} + ausinv\vec{\jmath} + u^2\vec{k}$ represents --------

   a. sphere b. elliptic paraboloid c. circle d. hyperbolic 1-sheet

8. $\int_C [fdx + gdy + hdz]$ is independent of path iff $fdx + gdy + hdz$ is----------

   a. 0 b. not exact c. 1 d. exact

9. $\int_0^1 \int_0^1 \int_0^1 xdxdydz =$ --------

   a. 1 b. 0 c. 2 d. 1/2

10. In triple function total mass of density $\sigma(x,y,z)$ in region R is given by M =---------

    a. $\int \int_R \sigma^2 dxdy$ b. $\int \int_R dxdy$ c. $\int \int_R \int \sigma dxdydz$ d. $\int \int_R \int 2\,\sigma dxdydz$

Q: 2 Do as directed: **[08]**

1. True or False: Beta function $\beta(m,n)$ is convergent for $m > 0, n > 0$.
2. True or False: If $\emptyset = \frac{1}{x}$ where $r^2 = x^2 + y^2$ then $\nabla^2\emptyset = 0$.
3. $\int_0^1 \int_0^x dxdy =$ --------------
4. For the curve $y = -x$, $\frac{ds}{dt} =$-------------
5. True or False: If $f = y^3$, $g = x^3 + 3xy^2$ then $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = 3x$
6. If $W = 2x^2 + y^2$ then $\nabla^2 W =$-------
7. A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f =$-------
8. True or False: If $\frac{\partial f}{\partial n} = \frac{\partial g}{\partial n}$ on *S* then $f - g$ is constant in *R*.

Q: 3 Answer in brief of the following questions. ***(Any Ten)*** ***[20]***

1. Find the equation of tangent plane and normal line to the surface $x^3y^2 - 3x^2z^3 = -zy + 2$ at the point (0, 2, 1)
2. In usual notation prove that $n\beta(m+1, n) = m\beta(m, n+1)$.
3. Let f be defined by $f(x, y, z) = x^2 siny + 1$. Find direction derivative of the function f at (0, 0, 0) in the direction of (1, 2, 3).
4. Evaluate $\int_C 3(x^2 + y^2)ds$, where C: $x^2 + y^2 = 1$ from (1,0) to (0,1) (clockwise direction)
5. Find the total mass of density of 1 in the region bounded by $y^2 = 6x$, $y = 0, x = 6$.
6. Change the order of an integration $\int_0^c \int_0^y f(x, y)dxdy$
7. Represent the surface $x^2 + y^2 + z^2 = a^2$ in parametric form.
8. Find the area of the region bounded by $r = a(1 + cos\theta)$
9. Evaluate: $\int_{(0,1,2)}^{(2\pi,0)} [((ydx + xdy)cosxy + dz)]$
10. Evaluate: $\int \int_S [xdydz + ydxdz + zdxdy]$ where $S: x^2 + y^2 + z^2 = a^2$ by applying Divergence theorem.
11. Let *R* be a closed region in space and S be its boundary. Let g be harmonic function in R, then prove that $\int \int_S \frac{\partial g}{\partial n} dA = 0$.
12. Evaluate $\int_C \bar{V}.\bar{t}\, ds$ using Stock's theorem for the given $\bar{V} = z\bar{\imath} + x\bar{\jmath}$ and $S: 0 \le x, y \le 1, z = 1$.

Q: 4 Attempt any Four of the following. **[32]**

(1) Define Beta and Gamma function. State and prove relation between Beta and Gamma function.

(2) In usual notation prove that: $curl(\bar{U} \times \bar{V}) = \bar{U} div\bar{V} - \bar{V} div\bar{U} + (\bar{V}.\bar{\nabla})\bar{U} - (\bar{U}.\bar{\nabla})\bar{V}$

(3) Transform $\int\int_R (x+y)^3 dxdy$ in uv-plane by taking $u = x + y, v = x - 2y$ where R is the parallelogram with vertices $(1,0),(0,1),(3,1)$ &$(2,2)$. Hence evaluate it.

(4) Find the centroid of density 1 in the plane area bounded by $y = 6x - x^2$ and $y = x$

(5) State and prove Green's theorem for plane.

(6) Verify both vector form (divergence and curl form) of Green's theorem for the given $\bar{V}$ and C, $\bar{V} = 7x\bar{\imath} - 3y\bar{\jmath}$, C: the circle $x^2 + y^2 = 4$.

(7) State and prove Divergence theorem of Gauss.

(8) Verify the Stock's theorem for $\bar{V} = (2x - y)\bar{\imath} - yz^2\bar{\jmath} - y^2z\bar{k}$; and surface *S*: the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

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