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**Sardar Patel University, Vallabh Vidyanagar**

B.Sc. - Semester-III : Examinations : 2022 [NC]

Subject : Mathematics

US03CMTH02

Max. Marks : 70

Numerical Analysis

Date: 18/06/2022, Saturday

Timing: 12.00 pm - 02.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

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Q: 1. Answer the following by choosing correct answers from given choices.

- [1] In usual notations, the formula  $\xi = x_{i+1} - \frac{(\Delta x_i)^2}{\Delta^2 x_{i-1}}$  is used by the method of  
 [A] False position [B] Bisection [C] Iteration [D] Aitken's  $\Delta^2$ -Process
- [2] Mid-points of intervals are used for approximation of root of an equation while using the method of  
 [A] False position [B] Bisection [C] Iteration [D] Aitkin's  $\Delta^2$ -Process
- [3] Which of the following intervals contains a root of  $x^2 - 3x - 4 = 0$   
 [A] [1, 3] [B] [3, 5] [C] [5, 7] [D] [7, 9]
- [4] If  $y_5 = 4$ , and  $y_{15} = 10$  then  $E^5 y_{10} =$   
 [A] 5 [B] 10 [C] 15 [D] 20
- [5]  $y_n - E^{-1} y_n =$   
 [A]  $\Delta y_{n+1}$  [B]  $\nabla y_{n+1}$  [C]  $\Delta y_n$  [D]  $\nabla y_n$
- [6] Which of the following is not true?  
 [A]  $y_{n+4} = E y_{n+3}$  [B]  $y_{n+4} = E^2 y_{n+2}$  [C]  $y_{n+4} = E^3 y_{n+1}$  [D]  $y_{n+4} = E^{-2} y_{n-2}$
- [7] For the given data 

x	$x_0 = 6$	$x_1 = 8$	$x_2 = 10$	$x_3 = 12$
y	10	14	20	30

,  $[x_2 \ x_3] =$   
 [A] 5 [B] 10 [C] 20 [D] 40
- [8] Langrange's Interpolation formula can be used for a data with \_\_\_\_\_ arguments.  
 [A] Rational [B] Irrational [C] only equally spaced [D] Unequally spaced
- [9] Runge-Kutta method is used for finding a numeric  
 [A] integral [B] derivative [C] solution of a differential equation [D] none
- [10] In usual notations, the formula

$$\int_a^b f(x).dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$$

is known as

- [A] Simpson's  $\frac{1}{3}$  rule [B] Simpson's  $\frac{3}{8}$  rule  
 [C] Trapezoidal rule [D] none

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false

- [1] For every approximation of root of an equation, mid-values of intervals are used in False position method. (True or False?).
- [2] Equation  $x^3 + 5x - 2 = 0$  has a root in the interval  $[0, 1]$ . (True or False?)
- [3] If  $\Delta y_1 = 1$  then  $y_4 = 4$  then  $E^3 y_1 = \dots$ .
- [4]  $\delta y_4 = 1$  then  $y_4 = 4$  then  $E^3 y_1 = \dots$ .
- [5] For 

x	1	4	5
y	8	12	15

 the divided difference  $[x_1, x_2]$  is  $\dots$ .
- [6] To use Lagrange's Interpolation formula for a data set  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$  it is necessary that  $x_0, x_1, \dots, x_n$  are equally spaced.
- [7] To use trapezoidal rule the number of subintervals must be even. (True or False?)
- [8] Runge-Kutta method is used to find numerical integral. (True or False?)

Q: 3. Answer any TEN of the following.

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- [1] Find an interval containing an initial approximation of  $\tan x = 1$
- [2] Find an interval containing an initial approximation of  $x^3 - 4x + 1 = 0$
- [3] Express  $\sin x = 5(x + 2)$  in the form of  $x = \phi(x)$ , so that the necessary condition for applying the Iteration method is satisfied.
- [4] If  $E^8 y_2 = 20$  and  $E^4 y_5 = 10$  then find  $\nabla y_{10}$
- [5] Prove that  $\mu = \frac{1}{2} (E^{\frac{1}{2}} + E^{-\frac{1}{2}})$
- [6] Prove that  $\Delta - \nabla = \delta^2$
- [7] If  $y_1 = 4, y_3 = 12, y_4 = 19$  and  $y_x = 7$  find  $x$ . Write the formula you use and also give it's name
- [8] For the given data 

x	$x_0 = 4$	$x_1 = 12$	$x_2 = 16$	$x_3 = 20$	$x_4 = 28$
y	10	14	20	30	45

 find  $[x_1, x_2] - [x_2, x_1]$
- [9] For the given data 

x	5	7	8
y	2	5	6

 find  $y(6)$
- [10] Discuss Euler's method for solving a differential equation.
- [11] Using Trapezoidal rule find  $\int_0^3 \cos x dx$ , with 3 subintervals of equal lengths.

[12] Using Trapezoidal rule find  $\int_0^5 \frac{1}{x+1} dx$ , with subintervals of length 1 unit.

Q: 4. Attempt ANY FOUR of the following questions.

[1] Using Bisection method find a real root of the equation  $x^3 - 4x - 9 = 0$  correct upto three decimal palaces

[2] Find a real root of  $\sin x = 10(x - 1)$  by iteration method correct upto three decimal places

[3] The populations of a town were as under

Year(x)	1891	1901	1911	1921	1931
Population (in thousand)	46	66	81	93	101

Estimate the population for the year 1895 and 1925

[4] Find the cubic polynomial which takes the following values

$$y(0) = 1, y(1) = 0, y(2) = 1, y(3) = 10$$

Hence find the value of  $y(4) - y(0.5)$

[5] Using Langrange's interpolation formula express the following function as a sum of partial fractions

$$\frac{x^2 + 6x + 1}{(x - 1)(x + 1)(x - 4)(x - 6)}$$

[6] Obtain 1<sup>st</sup> and 2<sup>nd</sup> order numerical differentiation formula from Newton's backward difference formula

[7] Evaluate  $\int_1^3 \frac{1}{x} dx$ , by using Simpson's  $\frac{1}{3}$  rule with 4 strips.

[8] Using Romberg's method, compute  $I = \int_0^1 \frac{1}{1+x} dx$ , correct upto three decimal places

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