Seat No.________ [70]

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - III (NC)) EXAMINATION - 2022
Friday, 17^{th} June 2022 MATHEMATICS: US03CMTH01
(ADVANCED CALCULUS)

Time : 12:00 Noon to 02:00 p.m. **Maximum Marks : 70**

Que.1 Fill in the blanks. 10

(1) For the curve $x^2 + y^2 = 1$, $\frac{ds}{dt}$ = ...
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) -1

(2) For $x + y = u$, $x - y = v$, jacobian J = ...
(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(3) If $f = y^3$, $g = x^3 + 3y^2x$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}$ = ...
(a) $3x^2$ (b) $3x^2 + 3y^2$ (c) $3y^2$ (d) $-3x^2$

(4) Parametric form of $x^2 + y^2 = z^2$ is $\bar{r}$ =................
(a) $u\cos v\bar{i} + u\sin v\bar{j} + u\bar{k}$ (b) $u\cos u\bar{i} + v\sin v\bar{j} + u\bar{k}$
(c) $u\cos v\bar{i} + v\sin u\bar{j} + v\bar{k}$ (d) $\cos v\bar{i} + \sin v\bar{j} + u\bar{k}$

(5) If $\bar{r} = u\cos v\ \bar{i} + u\sin v\ \bar{j} + u\ \bar{k}$ then $\bar{r}_u \cdot \bar{r}_u$ = ...
(a) 2 (b) 0 (c) 1 (d) u^2

(6) Area of plane region in Cartesian form is given by A = ...
(a) $\frac{1}{2}\int_C [x\ dx + y\ dy]$ (b) $\frac{1}{2}\int_C [x\ dy - y\ dx]$ (c) $\frac{1}{2}\int_C [x\ dy + y\ dx]$ (d) $\frac{1}{2}\int_C [x\ dx - y\ dy]$

(7) $\int_C [fdx + gdy + hdz]$ is independent of path iff *curl* $\bar{v}$ = ...,
where $\bar{v} = f\bar{i} + g\bar{j} + h\bar{k}$.
(a) $\bar{0}$ (b) 0 (c) $\bar{1}$ (d) 1

(8) $xdx + ydy + zdz$ = ...
(a) $d\left[\frac{x^2+y^2+z^2}{2}\right]$ (b) $d\left[\frac{x^2+y^2+z^2}{3}\right]$ (c) $d\left[x^2+y^2+z^2\right]$ (d) $d\left[\frac{(x+y+z)^2}{2}\right]$

(9) $\int_0^2\int_0^x dydx$ = ...
(a) 1 (b) 1/2 (c) x (d) 2

(10) $\int_0^1\int_0^2\int_0^3 dxdydz$ = ...
(a) 1 (b) 6 (c) 3 (d) 2

Que.2 Write TRUE or FALSE. [8]

(1) A function $f(x, y, z)$ is said to be harmonic if $\nabla^2 f = 0$.

(2) For the curve $y = -x$, $\frac{ds}{dt} = -\sqrt{2}$

(3) The first form of Green's theorem is $\iiint_R [f\bar{\nabla}^2 g + \bar{\nabla} f \cdot \bar{\nabla} g]dV = \iint_S g\frac{\partial f}{\partial n}\, dA$

(4) Parametric form of $x^2 + y^2 = z$ is $\bar{r} = \sqrt{u}\cos v\bar{i} + \sqrt{u}\sin v\bar{j} + u\bar{k}$

(P.T.O.)

(5) If $f = -xy^2$, $g = x^2y$ then $\dfrac{\partial f}{\partial y} - \dfrac{\partial g}{\partial x} = -4xy$

(6) $\int_0^1 \int_0^1 \int_0^1 x\, dxdydz = \dfrac{1}{2}$

(7) In Divergence theorem , $\iiint_R \bar{\nabla} \cdot \bar{u}\, dv = \int_S \bar{u} \cdot \bar{n}\, dA$

(8) $\dfrac{ds}{dt} = \dfrac{d\bar{r}}{dt}$

Que.3 Attempt the following (Any TEN) [20]

(1) Evaluate $\int_0^{\pi/2} \int_0^1 x^2y^2\, dydx.$

(2) Evaluate $\int_0^1 \int_{x^2}^{x} (1 - xy)\, dydx.$

(3) Evaluate $\int_0^2 \int_0^y e^{x+y}\, dxdy.$

(4) Show that in $\int_{(0,1,2)}^{(2,\pi,0)} [(ydx + xdy)\cos xy + dz]$ the form under integral sign is exact .

(5) Show that in $\int_{(1,1,2)}^{(3,-2,-1)} [yzdx + xzdy + xydz]$ the form under integral sign is exact .

(6) Represent the surface $\bar{r} = a\cos v \cos u\bar{i} + a\cos v \sin u\bar{j} + a\sin v\bar{k}$ in cartesian form

(7) Represent the surface $x^2 + y^2 + z^2 = a^2$ in parametric form .

(8) Represent the surface $x + y + z = 5$ in parametric form .

(9) Represent the surface $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = z^2$ in parametric form .

(10) Evaluate $\int_1^2 \int_{-1}^1 \int_0^1 xy^2z^2\, dxdydz$.

(11) Evaluate $\int_0^1 \int_{-2}^2 \int_{-1}^1 x^2y^2z^3\, dxdydz$.

(12) Evaluate triple integral of x^2 over the region bounded by the plane $x = y = z = 0$; $x+y+z = 1$.

Que.4 Attempt the following (Any FOUR) [32]

(1) Find the coordinate $\bar{x}$ of centroid of density 1 in the plane area bounded by $y = 6x - x^2$ & $y = x$.

(2) Transform $\iint_R (x^2 + y^2)\, dxdy$ in uv-plane by taking $x + y = u, x - y = v$.Then evaluate it, where R:Parallelogram with vertices (0,0),(1,1),(2,0),(1,-1).

(3) State and prove Green's theorem for plane.

(4) Change the order of integration in $\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y)\, dydx.$

(5) State and prove divergence theorem of Gauss.

(6) Find moment of inertia of surface $S : x^2 + y^2 = z^2; 0 \le z \le h.$ of density 1 about z-axis .

(7) Evaluate $\int_C \overline{V} \cdot \bar{t}\; ds$ by Stoke's theorem , where $\overline{V} = z\bar{i} + x\bar{j} + y\bar{k}$ and S : the square with vertices $(0, 0, 0), (1, 0, 0), (0, 1, 0)(1, 1, 0)$.

(8) By using triple integral , find volume of the region bounded by R : the tetrahedral cut from the first octant by the plane $3x + 4y + 2z = 12$.

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