SARDAR PATEL UNIVERSITY
F. Y. B. Sc. SEMESTER-2
ANALYTIC SOLID GEOMETRY
US02CMTH01
MATHEMATICS

Date: 30-09-2022
Friday

Time: 09:30 am to 11:30 am
Total Marks: **70**

Q:1 Answer the following by selecting correct choice from the given options. (10 marks)

(1) Distance between any point on a sphere $(x-1)^2 + (y-1)^2 + (z-1)^2 = 9$ and its Centre is _____.

   (a) 1   (b) 3   (c) 9   (d) none of these

(2) Intersection of sphere and plane is _____.

   (a) Line   (b) Plane   (c) Circle   (d) Ellipse

(3) The necessary and sufficient condition of two spheres touch each other externally is _____.

   (a) $C_1C_2 > r_1 + r_2$ (b) $C_1C_2 = r_1 + r_2$ (c) $C_1C_2 = |r_1 - r_2|$ (d) $C_1C_2 < r_1 + r_2$

(4) The surface $\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = by$; represents a _____.

   (a) Elliptic paraboloid   (b) hyperbolic paraboloid

   (c) Elliptic hyperboloid of one sheet   (d) none of these

(5) The major axis of the surface $-\dfrac{x^2}{25} + \dfrac{y^2}{9} + \dfrac{z^2}{16} = 1$ is _____.

   (a) X   (b) Y   (c) Z   (d) None of these

(6) Line intersect a cone in maximum _____ point.

   (a) three   (b) one   (c) two   (d) None of these

(7) Every cone is generated by _____.

   (a) Lines   (b) Circle   (c) Ellipse   (d) Parabola

(8) Every cone has _____ vertex.

   (a) Only one   (b) finitely many   (c) Two   (d) infinitely

(9) A cylinder is a surface generated by a straight line which is always _____ to a fixed line

   (a) parallel   (b) perpendicular   (c) above   (d) below

(10) Every cylinder has _____ generators.

   (a) only one   (b) finitely many   (c) two   (d) infinitely many

Q:2 True or False   **(8 marks)**

(1) Centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ is $(1, -2, 3)$.

(2) Sphere $x^2 + y^2 + z^2 - 6x + 8y - 10z = 0$ is passing through the point $(0,0,0)$.

(3) Intersection of surface with XY-plane gives XY-trace.

(4) Vertex of second degree homogeneous equation of cone is $(0,0,0)$.

(5) Given fixed curve in the cone is called generator.

(6) In a cylindrical polar co-ordinate system the equation Z=2 represents a plane parallel to ZX-plane.

(7) Every plane section of a right circular cylinder by a plane perpendicular to its axis is Parabola.

(8) If $Ax^2 + By^2 + Cz^2\ 2Fyz +2Gzx + 2Hxy = 0$ is reciprocal cone of $ax^2 + by^2 + cz^2 = 0$ then $F = 1$.

Q:3 Answer the following questions in short. (Any Ten) (20 Mark)

(1) Find the equation of sphere whose diameter is the line segment joining (3,4,5) and (1,2,3).

(2) Find radius and centre of the sphere $x^2 + y^2 + z^2 -2x - 2y - 2z = 6$.

(3) Find the equation of tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z -24 = 0$ at the point (1,1,2).

(4) Discuss the symmetry of $\dfrac{y^2}{9} + \dfrac{z^2}{4} = 5x$.

(5) Find the Jacobian for $u = x + 2y$, $v = 3x + y$.

(6) Plot the point $(3,40^0,60^0)$ in R^3.

(7) Define tangent line and tangent plane to the cone.

(8) Find the equation of cone whose vertex is (0,0,0) and base is $ax^2 + by^2 = 1$, $z = 2$.

(9) Find the equation of cone with vertex at origin and passes through the curve $ax^2 + by^2 = 2z$, $lx + my + nz = p$.

(10) Find the equation of a right circular cylinder of radius 2 and whose axis is Z-axis.

(11) Find the equation of the cylinder with the guiding curve given by $x^2 + 2y^2 = 1$, $z = 0$ and generators parallel to $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$.

(12) Define Right circular cylinder.

Q:4 Attempt <u>any Four</u> questions of the following. (32 marks)

(1) Show that the plane $lx + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ if and only if $(l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d) = (ul + vm + wn + p)^2$.

(2) Find the centre and radius of the circle $S: x^2 + y^2 + z^2 = 4$; $P: x + y + z = 1$.

(3) Identify, describes and sketch the surface $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$; ($b \geq a,c$).

(4) Identify and describes the surface $\dfrac{y^2}{4} - \dfrac{z^2}{1} = 2x$.

(5) Find the equation of cone with vertex (α, β, γ) and whose generators touch the sphere $x^2 + y^2 + z^2 = a^2$.

(6) Find equation of cone with vertex is (α, β, γ) and guiding curve is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $z = 0$.

(7) Prove that the reciprocal cone of the reciprocal cone is the cone itself.

(8) Find equation to the cylinder whose generator touch the sphere $x^2 + y^2 + z^2 = a^2$ and are parallel to the line to $\dfrac{x}{l} = \dfrac{y}{m} = \dfrac{z}{n}$.

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