[S1]

SARDAR PATEL UNIVERSITY

B.Sc. (II-Semester) (NC) EXAMINATION 2022

Monday, 2ⁿᵈ May

12:00pm-02:00pm

US02EMTH02-Mathematics

MATHEMATICS

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: [10]

1. $\lim_{x \to 1} \left[\frac{1}{x-1} + \frac{1}{1-x} \right] =$ -----------

    a. 1   b. 0   c. 2   d. doesn't exist

2. $\frac{d}{dx}(logx) =$ -----------

    a. $\frac{1}{x^2}$ b. $\frac{1}{x}$ c. $\frac{1}{logx}$ d. e^x

3. $\frac{d}{dx}(u.v) =$ ----------- if u and v are functions of x.

    a. $u\frac{du}{dx} - u\frac{dv}{dx}$ b. $\frac{\frac{dv}{dx}}{u^2}$ c. $u\frac{dv}{dx}$ d. $u\frac{dv}{dx} + v\frac{du}{dx}$

4. The integration is also know --------------

    a. subtraction   b. combination   c. antiderivative   d. none of these

5. The $\int logxdx$ can be found by using integral by parts, taking u and v as

    a. $u = 1, v = logx$ b. $u = logx, v = 1$ c. $u = logx, v = x$ d. none of these

6. $\int sec^2xdx =$ ---------

    a. $cotx$ b. $tanx$ c. $-cotx$ d. $-tanx$

7. $\int_{-a}^{a} x^2dx =$ -------------

    a. $\frac{2}{3}$ b. 0   c. $\frac{2}{3} a^3$ d. none of these

8. $\int_{log2}^{log5} e^xdx =$ --------

    a. -3   b. 3   c. 0   d. 7

9. The general solution of the differential equation $\frac{dy}{dx} = 1$ is --------

    a. $y = x$ b. $y = x^2 + c$ c. $y = x + c$ d. none of these

10. The order and degree of differential equation $\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$

    a. 1, 2   b. 2, 1   c. 3, 3   d. 2, 2

(1)

[P. T. O.]

Q: 2 Do as Directed: [08]

1. True or False: If f is an even function then $f(-x) = f(x)$

2. True or False: $\frac{d}{dx}(10^3) = 0$

3. If $\int f(x)dx = sinx$ then $\int xf(x)dx = $ --------------

4. For any point circle $x^2 + y^2 = a^2$ then $\int(x^2 + y^2)dx = $ -------------

5. True or False: Primitive of $sinx$ is $cosx$.

6. True or False: $\int_{-\pi}^{\pi} \frac{1}{\pi} sinxdx = \frac{2}{\pi}$

7. The solution of nth order differential equation have -----------arbitrary constants.

8. The differential equation representing all lines of family $y = mx + c$ is ---------------

Q:3 Answer in brief of the following questions. *(Any Ten)* [20]

1. Find $\lim_{x \to 1} \frac{x^6 - 1}{x^{15} - 1}$ $x \in R - \{1\}$

2. Find $\lim_{x \to 0} \frac{sin5x - sin7x}{sinx}$

3. Find derivative of $x^3 + y^3 = 3axy$

4. Evaluate: $\int(sinx + e^x + 4^x + x^4)\ dx$

5. Evaluate: $\int x\ cosx\ dx$

6. Evaluate: $\int \frac{1}{4x^2 + 9}dx$

7. State the fundamental principle of definite integration.

8. Find the value of $\int_0^\pi [af(x) + bg(x)]dx$ where $\int f(x)dx = cosx$ $\int g(x)dx = sinx$

9. Prove that $\int_1^e logxdx = 1$

10. Define: Differential Equation, Order and Degree of Differential Equation.

11. Verify that $y = e^x$, $x \in R$ is a solution of the differential equation $\frac{dy}{dx} = y$

12. Solve: $xdy - ydx = 0$

Q:4 Attempt any Four of the following. [32]

1. (i) If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$, $n \in N$ then find n.

   (ii) Evaluate: $\lim_{x \to 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$

2. Find $\frac{dy}{dx}$ for the given: (i) $y = cos^{-1}\left(\frac{3 + 5cosx}{5 + 3cosx}\right)$

   (ii) $y = x^x$

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3. Evaluate: (i) $\int xe^x\,dx$

(ii) $\int \frac{x^2}{1+x^6}\,dx$

4. Evaluate: (i) $\int \frac{a+b\cos x}{\sin^2 x}\,dx$

(ii) $\int \sqrt{1-\cos x}\,dx \quad 0 < x < \pi$

5. Evaluate: (i) $\int_0^{\frac{\pi}{4}} \frac{1}{4\sin^2 x + 5\cos^2 x}\,dx$

(ii) $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}\,dx$

6. Evaluate: (i) If $\int_0^k \frac{\tan x}{1+\tan x}\,dx = \frac{\pi}{4}$ then find the value of k.

(ii) $\int_0^\pi \sin^4 x \cos^3 x\,dx$

7. Find the differential equation of the following family of the curves, where a and b are arbitrary constant.

(i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(ii) $y = a\sin(b + x)$

8. Solve: (i) $y(1 + e^x)dy = e^x(1 + y)dx$

(ii) $xy\frac{dy}{dx} = y + 2$