SEAT No.________

[54-E]

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester- II : Examination : 2021-22 [NC]

Subject : Mathematics US02CMTH02 Max. Marks : 70

Matrix Algebra and Differential Equations

Date: 26/04/2022, Tuesday Timing: 12.00 pm - 02.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] The elements of principal diagonal of a skew symmetric matrix are equal to
[A] -1 [B] 0 [C] 1 [D] none

[2] For a square matrix A over R the matrix $A - A'$ is
[A] symmetric [B] skew symmetric [C] Hermitian [D] skew Hermitian

[3] If $P = \begin{bmatrix} 4 & -1 \\ 3 & 5 \end{bmatrix}$ then $PP' =$
[A] $\begin{bmatrix} 17 & 7 \\ 7 & 34 \end{bmatrix}$ [B] $\begin{bmatrix} -17 & 7 \\ 7 & -34 \end{bmatrix}$ [C] $\begin{bmatrix} 7 & 34 \\ 17 & 7 \end{bmatrix}$ [D] I

[4] If λ is a characteristic root of A then the matrix $A - \lambda I$ is
[A] orthogonal [B] non-singular [C] singular [D] Hermitian

[5] If $|A + 4I| = 0$ then one of the characteristic roots of A is
[A] 0 [B] -4 [C] 4 [D] 1

[6] If 3 is a characterstic root of A then
[A] $|I + 3A|$=0 [B] $|I - 3A|$=0 [C] $|A + 3I|$=0 [D] $|A - 3I|$=0

[7] $\frac{1}{D-1}e^{-x} =$
[A] $\frac{1}{2}e^{-x}$ [B] $-\frac{1}{2}e^{-x}$ [C] $\frac{x}{2!}e^{-x}$ [D] $-\frac{x}{2!}e^{-x}$

[8] Complementary function of $(D^2 - 4D + 4)y = e^x$ is
[A] $(c_1x + c_2)e^{2x}$ [B] $(c_1 + c_2)e^{2x}$
[C] $e^{2x}(c_1 \cos 2x + c_2 \sin 2x)$ [D] $c_1 \cos 2x + c_2 \sin 2x$

[9] $\frac{1}{D^2 + m^2} \cos mx =$
[A] $\frac{x}{2m} \sin mx$ [B] $-\frac{x}{2m} \sin mx$ [C] $-\frac{x}{2m} \cos mx$ [D] $\frac{x}{2m} \cos mx$

[10] $\frac{1}{D^4 + D^2 + 1} \cos 2x =$
[A] $\frac{1}{10} \cos 2x$ [B] $-\frac{1}{10} \cos 2x$ [C] $-\frac{1}{13} \cos 2x$ [D] $\frac{1}{13} \cos 2x$

(P. T. O.)

Q: 2. In the following, depending on the type of question, either fill in the blank or answer whether a statement is true false. 8

[1] Traspose of $\begin{bmatrix} 5 & 5-i \\ 4i & -2 \end{bmatrix}$ is ______.

[2] Determinant value of $\begin{bmatrix} 2 & 5 \\ 8 & -8 \end{bmatrix}$ is ______.

[3] Matrix $\begin{bmatrix} 5 & 1 & -7 \\ 2 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ is a non-singular matrix. (TRUE/FALSE?)

[4] $\begin{bmatrix} 2 & -1 & 5 \\ 3 & 2 & -1 \\ 1 & 3 & 4 \end{bmatrix}$ is a symmetric matrix. (TRUE/FALSE?)

[5] $\frac{1}{(D-5)^3}e^{7x} =$ ______.

[6] Complementery function for $(D^3 - D^2 - 6D)y = e^x - e^{-x}$ is ______.

[7] Particular solution of $D^4 y = x^5$ is ______ .

[8] Particular solution of $(D^3)y = \sin 2x$ is ______ .

Q: 3. Answer ANY TEN of the following. 20

[1] Define :(i) Transpose of a Matrix (ii) Unit Matrix

[2] If A is Hermitian then prove that $B^\theta AB$ is Hermitian.

[3] Determine whether $\begin{bmatrix} 7-4i & 5-i & 1 \\ 4i-1 & 6+i & 2-i \\ 3 & i-4 & 9+4i \end{bmatrix}$ is Skew-Hermitian or not.

[4] Find the characterstic roots of $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$.

[5] Determine whether the matrix $A = \begin{bmatrix} 4 & 1 \\ 5 & -5 \end{bmatrix}$ is orthogonal or not.

[6] Determine whether $\begin{bmatrix} -3 & 4 & 0 \\ 8 & -1 & 7 \\ 1 & 3 & 0 \end{bmatrix}$ is singular or non-singular.

[7] Find $\frac{1}{(D+2)^3}e^{-2x}$.

[8] Find the complementary function of $(D^2 - 8D + 16)y = e^{2x}$.

[9] Find the particular integral of $(D-1)^5 y = e^{11x}$.

[10] Find the complementary function of $(D^3 - 3D^2)y = x \sin x$.

[11] Find the particular integral of $(D^4 + D^2)y = \sin 4x$.

[12] Find the complementary function of $(5 - 2D)^2 y = \cos 2x$.

Q: 4. Attempt ANY FOUR of the following. **32**

[1] Prove that every square matrix can be expressed in one and only one way as a sum of a symmetric and a skew-symmetric matrix.

[2] For $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$, where $l = \frac{1}{\sqrt{2}}$, $m = \frac{1}{\sqrt{6}}$ and $n = \frac{1}{\sqrt{3}}$ show that $AA' = I$.

[3] State and prove *Cayley-Hamilton theorem*.

[4] Show that the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ satisfies *Cayley-Hamilton* theorem. Hence or otherwise obtain A^{-1}.

[5] Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$ where m is a constant.

[6] Solve : $(D^3 - 5D^2 + 7D - 3)y = \cosh x$.

[7] In usual notations prove that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$, where V is a function of x.

[8] Solve : $(D^2 + 9)y = x \sin x$.

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