SARDAR PATEL UNIVERSITY
VALLABH VIDYANAGAR

SYLLABUS EFFECTIVE FROM: 2017-18
Syllabus for M.Sc. (Mathematics)
Semester II

There will be six courses, each of 4 credits. The session work for each course will comprise of four lectures each of one hour duration and if needed, based on the strength of the class, one hour per week for seminar. For a stipulated period of certain weeks, decided by the department/centre for each course separately, one hour out of the four allotted hours, may be reallocated to seminars. There will be a 1 credit course for comprehensive viva and Mathematics presentation for which students may seek guidance from concerned teachers. Thus a student will be provided 30 hours actual teaching per week; and he/she will be required to earn 25 credits during the semester. Each course will have a weighting of 100 marks (70 marks for University examination + 30 marks for inter assessment. Internal assessment will comprise of 1 internal test of 20 marks, a seminar of 5 marks and quizzes of 5 marks). Each student will take 6 courses in consultation and with approval of the department. There will be 5 core courses and 1 elective course to be taken by a student.

Viva: There will be a viva-voce examination of 50 marks at the end of each semester covering all the courses offered during the semester.

List of courses

Core Courses

PS02CMTH21: Real Analysis I
PS02CMTH22: Algebra I
PS02CMTH23: Differential Geometry
PS02CMTH24: Functional Analysis I
PS02CMTH25: Methods of Partial Differential Equations
PS02CMTH26: Comprehensive Viva

Elective Courses

PS02EMTH21: Graph Theory I
PS02EMTH22: Mathematical Classical Mechanics
PS02EMTH23: Number Theory
PS02EMTH25: C Programming and Mathematical algorithms I

PS02CMTH21: Real Analysis I

Unit I Algebra and σ-algebra of sets, Borel sets in $\mathbb{R}$, Lebesgue outer measure in $\mathbb{R}$, measurable sets and Lebesgue measure on $\mathbb{R}$, non-measurable set, measurable functions.

Unit II Littlewood's three principles, Egoroff's theorem, the Lebesgue integral of a measurable simple function vanishing outside a set of finite measure, the Lebesgue integral of a bounded function over a set of finite measure, comparison of Riemann and Lebesgue integral, bounded convergence theorem. Lebesgue integral of a nonnegative measurable function.

Unit III Fatou's lemma and monotone convergence theorem, Beppo-Levis theorem, general Lebesgue integral, dominated convergence theorem, convergence in measure, relation with convergence a.e.

Unit IV Vitali's theorem (statement only), functions of bounded variation, Jordan's lemma, differentiation of an integral, continuity and bounded variation of indefinite integral, absolute continuity of indefinite integral, different forms of fundamental theorem of integral calculus, relation between indefinite integral and absolute continuity.

Textbook

1 Royden H.L., Real Analysis (Third Edition) Mac Millan, 1998.

Sections: 1.4, 2.7, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4.1, 4.2, 4.3, 4.4, 5.1 (without proofs), 5.2, 5.3, 5.4.

Reference Books

1 Rana, I. K., An Introduction to Measure and Integration, Narosa Publ. House, New Delhi, 1997.

2 De Barra G, Introduction to Measure Theory, Van Nostrand Reinhold Co., 1974.

PS02CMTH22: Algebra I

Unit I Definitions of a ring, integral domain, division ring, field, examples of field, characteristic of an integral domain, pigeonhole principle, equivalence relation on an integral domain, equivalence classes $[a, b]$, Field of quotients of an integral domain, Euclidean ring, ideals of Euclidean ring, existence of unit element in a Euclidean ring, units, the relation of division in an integral domain, gcd and its existence in a Euclidean ring, associates, prime and relatively prime elements in Euclidean ring, unique factorization theorem in a Euclidean ring, characterization of maximal ideals in a Euclidean ring, definitions of polynomial rings over Z, Q and a field, $F[x]$ is a Euclidean ring (only statement), definition of an irreducible polynomials and example of reducible and irreducible polynomials.

Unit II Extension field and its examples, degree of an extension field, finite extension, algebraic and transcendental elements of an extension field, smallest extension containing a particular element, and its application to characterization of algebraic element, degree of algebraic element over a field, sum and product of algebraic elements, algebraic extension, algebraic complex number, roots of polynomials in an extension field, remainder theorem, bound on number of roots of a polynomial over a field, existence of a root of a polynomial, existence of all roots of a polynomial.

Unit III Splitting field, existence of a splitting field, isomorphism from one field F onto another field F', its extension as a ring homomorphism from $F[x]$ onto $F'[t]$, its factorization as an isomorphism from $F[x]/f(x)$ onto $F'[t]/f'(t)$, isomorphism mapping one root of an irreducible polynomial onto another root of a polynomial, uniqueness of splitting field, derivative of a polynomial and its arithmetic, multiple roots of a polynomial and characterization of multiple roots of an irreducible polynomial over a field of characteristic zero and nonzero, simple extension, simplicity of a finite extension of a field of characteristic zero, automorphism of a field, fixed field of a group of automorphisms. Bound on order of $G(K,F)$ for a finite extension K of F.

Unit IV Field of rational functions in n variables over a field, elementary symmetric functions, field of symmetric rational functions over a field, its characterization as the field generated by elementary symmetric functions, normal extension and its characterization as a splitting field, automorphism of a splitting field, mapping one root to another, Galois group, fundamental theorem of Galois Theory, solvable group, characterization of solvability of symmetric groups of permutations on n symbols, field containing all n^{th} root of unity for some n and commutativity of splitting field of $x^n - a$, solvability of a polynomial by radical implying solvability of its Galois group, radical extension of a field, Abel's theorem.

Textbook

1. Herstein, I.N., Topics in Algebra, Wiley Eastern. Ltd., New Delhi, 1975.

   3.1(Definitions only), 3.2(Definitions and Lemma 3.3.2 only), 3.6, 3.7, 3.9 (review without proof only), 3.10 (review without proof only), 5.1, 5.3, 5.5, 5.6, 5.7

Reference Books

1. Gallian, J., Contemporary Abstract Algebra, (Eight Edition), Books/Cole Cengage Learning, Belmont, 2013.
2. Fraleigh J.B., A First Course in Abstract Algebra, (Seventh Edition), Addison-Wesley, 2003.
3. Dummit, D.S. and Foote, R.M., Abstract Algebra, (Third Edition), John Wiley & Sons Inc., 2004.

PS02CMTH23: Differential Geometry

Unit I	Space curves, planar curves, parameterization, curvature, torsion, signed curvature, Frenet-Serret equations, fundamental theorem of curve theory, isoperimetric inequality.
Unit II	Surfaces: smooth surfaces, tangents, normals, first fundamental form, isometries of surfaces, conformal mappings of surfaces, surface area.
Unit III	Second fundamental form, Gauss map, normal and principal curvature, geodesic curvature and normal curvature of a curve, Meunier's theorem, Euler's theorem, Gaussian and mean curvature.
Unit IV	Tangent vector field and its covariant derivative, Gauss equations, Christoffel symbols, geodesics, geodesic equations, characterization of geodesics on surfaces like sphere, cylinder, plane and surface of revolution, Codazzi-Mainardi equations, Theorema Egregium, local Gauss Bonnet theorem (statement only) and its applications.

Textbook

1 Pressley Andrew, Elementary Differential Geometry, SUMSeries, (Second Edition), 2010.

Chapter 1: (Except sections 1.4 and 1.5), Chapter 2, Chapter 3: 3.1 (Except 3.1.4), 3.2, Chapter 4: 4.1, Definition 4.2.1 and examples, definition of a smooth map, 4.4, definition of a normal, Chapter 6: 6.1, 6.2 (except 6.2.4, 6.2.5), 6.3, 6.4.1, 6.4.2, Chapter 7: 7.1, 7.2, 7.3, 7.4 (except 7.4.6, 7.4.7, 7.4.8, 7.4.9, 7.4.10), Chapter 8: 8.1, 8.2, Chapter 9: 9.1, 9.2, 9.3.1, Chapter 10: 10.1, 10.2.1, Chapter 13: 13.1.2 (statement and applications)

Reference Books

1 Goetz A., Introduction to Differential Geometry, Addison Wesley, Publ. Co., 1970.

2 Weatherburn C.E., Differential Geometry in Three Dimensions, Cambridge University Press, 1964.

PS02CMTH24: Functional Analysis I

Unit I Inner product spaces, normed linear spaces, Banach spaces, examples of inner product spaces, Polarization identity, Schwarz inequality, parallelogram law, uniform convexity of the norm induced by inner product, orthonormal sets, Pythagoras theorem, Gram-Schmidt othonormalization, Bessel's inequality, Riesz-Fischer theorem. Hilbert spaces, orthonormal basis, characterization of orthonormal basis, separable Hilbert spaces.

Unit II Uniqueness of best approximation from a convex subset of inner product space to a point, orthogonality and best approximation, Gram matrix and its applications, existence and uniqueness of best approximation from a convex subset of a Hilbert space to a point, continuity of a linear mapping, projection theorem and Riesz representation theorem, reflexivity of a Hilbert space. Unique Hahn–Banach extension theorem, weak convergence and weak boundedness.

Unit III Bounded operators, equivalence of boundedness and continuity of an operator, boundedness of the operator associated to an infinite matrix, adjoint of a bounded operator, properties of adjoint, relations between zero space and the range of operators, normal, unitary and self-adjoint operators, examples, characterizations and results pertaining to these operators, positive operators and generalized Schwarz inequality.

Unit IV Spectrum, eigenspectrum, approximate eigenspectrum, definition and characterization, spectrum of a normal operator, numerical range, relations of numerical range and different spectra, spectral theorem for a normal/self-adjoint operator on a finite dimensional Hilbert space, compact operators, properties of compact operators, Hilbert-Schmidt operator and its properties, spectrum of a compact operator, spectral theorem for a compact self-adjoint operator.

Textbook

1 Limaye B.V., Functional Analysis, New Age International Publ. Ltd., New Delhi, 1996.

Chapter 6: Sections 21, 22, 23, 24, Chapter 7: Sections 25, 26, 27, 28.

Reference Books

1 Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.
2 Thumban Nair, Functional Analysis: A First Course, Prentice-Hall of India, New Delhi, 2002.

PS02CMTH25: Methods of Partial Differential Equations

Unit I Origin of second order partial differential equations, linear second order partial differential equations with constant coefficients, solutions for f(x, y) to be polynomial, exponential, sin/cos functions, general method for homogeneous equations.

Unit II Second order partial differential equations with variable coefficients, solution by method of changing variables u = logx, v = logy for special type of equations, non-linear first order partial differential equations: compatible system of first order partial differential equations, solution by Charpit's method and Jacobi's method.

Unit III Classification of second order partial differential equations and canonical form, non-linear second order partial differential equations: solution by Monge's method, special case and general case.

Unit IV Separation of variable method: solution of three special equations – Laplace, wave and diffusion equation, solution of these equations in different coordinate systems. Boundary Value Problems (BVP): Dirichlet and Neumann BVP, solutions of BVP for a circle and a rectangle.

Textbooks

1 Amarnath T., Elementary Course in Partial Differential Equations, Narosa Pub. House, New Delhi, 1997.
(Chapter 2: Section 2 (2.4.6 to 2.4.9))

2 Sneddon I. N., Elements of Partial Differential Equations, McGraw- Hill Pub. Co., 1957.
Chapter 2: Section 9,10,11,13 and Chapter 3: Section 1,4,5,9,11

Reference Books

1 Grewal B. S. and Grewal, J. S., Higher Engineering Mathematics, Khanna Pub., New Delhi, 2000.

2 Raisinghania M. D., Advanced Differential Equations, S. Chand & Co., 1995.

3 Phoolan Prasad and Ravindran R., Partial Differential Equations, Wiley Eastern.

PS02EMTH21: Graph Theory I

Unit I Review of basic facts about graphs: connected graph, distance and diameter, tree, Euler graph, fundamental circuits, matrix representation of graphs, isomorphic graphs.

Directed Graphs: definitions and examples, vertex degrees, some special types of digraphs, directed path and connectedness, Euler digraphs.

Unit II Trees with directed edges, spanning out-tree, spanning in-tree and their relation with Euler digraph, incidence matrix, circuit matrix and adjacency matrix of digraphs, fundamental circuits and fundamental circuit matrix in digraphs.

Unit III Chromatic number, chromatic partitioning, uniquely colorable graphs, chromatic polynomial, four-color problem. Hamiltonian cycles: necessary conditions, sufficient conditions.

Unit IV Matching and covers: maximum matching, Hall's matching condition, min-max theorems, independent sets, vertex cover, edge cover.

Textbooks

1 Deo Narsingh, Graph Theory with Applications to Engg. and Computer Science, Prentice-Hall of India Pvt. Ltd., New Delhi, 1999.
Chapter 9: Sections 9.1 to 9.9 (Except 9.3, Kirchhoff matrix from 9.9), Chapter 8: Sections 8.1 to 8.3 (Except dominating sets, 8.6)

2 Douglas B, West, Introduction to Graph Theory, Pearson Education, Inc. 2002.
Chapter 3: Section 3.1 (up to 3.1.24), Chapter 7: Section 7.2 (up to 7.2.8) & 7.2.19

Reference Books

1 Clark J. and Holton D. A., A First Look at Graph Theory, Allied Publishing Ltd., 1991.

2 Wilson Robin J., Introduction to Graph Theory, Pearson Education Asia Pvt. Ltd., 2000.

PS02EMTH22: Mathematical Classical Mechanics

Unit I	Constraints and their classification, principle of virtual work, de'Almbert's principle, various forms of Lagrange's equations of motion for holonomic systems, examples.
Unit II	Euler-Lagrange equations in various forms (statements only), Hamilton's variational principle, derivation of Lagrange's equation from Hamilton's variational principle, generalized momentum, cyclic coordinates, general conservation theorem, conservation of linear momentum and angular momentum in Lagrangian formalism and symmetry properties, energy function and conservation of total energy in Lagrangian formalism.
Unit III	Hamilton's canonical equation of motion, relation with Lagrange's equation, cyclic coordinate, Routhian procedure, variational principle approach to Hamilton's equation of motion, examples.
Unit IV	Canonical transformations, generating functions, symplectic condition, infintesimal canonical transformations, examples. Poisson bracket, Lagrange bracket, formal solution of equations of motion in terms of Poisson brackets, examples.

Textbook

1 Goldstein H., Poole C. and Safko J., Classical Mechanics, (Third Edition), Pearson Education, Inc., Indian Low Price Edition, 2002.
Articles: 1.3, 1.4, 1.5 and 1.6, 4.1 (understanding of constraints and generalized coordinates in a rigid body motion); 2.1, 2.2 (statements only), 2.3, 2.6 and 2.7; 8.1,8.2, and 8.5; 9.1,9.2, 9.4, 9.5 and 9.6.

Reference Books

1 Bhatia V. B., Classical Mechanics, Narosa Publishing House, 1997.
2 Sankara Rao K., Classical Mechanics, Prentice-Hall of India, 2005.

PS02EMTH23: Number Theory

Unit I	The division algorithm, the greatest common divisor, the Euclidean algorithm, the fundamental theorem of arithmetic, infinitude of prime numbers (Euclid's proof), basic properties of congruence, linear congruences and the Chinese remainder theorem.
Unit II	Fermat's little theorem, Wilson's theorem, the sum and number of divisors, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, some properties of the phi-function.
Unit III	Euler's criterion, Legendre's symbol: definition and its properties, evaluation of $(-1|p)$ and $(2|p)$, Gauss lemma, quadratic reciprocity.
Unit IV	Algebraic numbers, Algebraic integers, quadratic fields, units and primes in quadratic fields, unique factorization.

Textbooks

1 Burton David M., Elementary Number Theory, (Seventh Edition) McGraw Hill Education.

2.2, 2.3, 2.4, 3.1, 3.2, 4.2, 4.4, 5.2 (except theorems 5.2, 5.3), 5.3, 6.1, 6.2, 7.2, 7.3, 7.4, 9.1, 9.2, 9.3

2 Nivan Ivan, Zuckermann H. S. and Montgomery H. L., An Introduction to the Theory of Numbers, (Fifth Edition) John Wiley & Sons Inc.

9.2, 9.4, 9.5, 9.6, 9.7, 9.8

Reference Books

1 Apostol Tom M., Introduction to Analytic Number Theory, Springer.
2 Baker Alan, A Concise Introduction to the Theory of Numbers, (Cambridge Uni. Press, Cambridge).
3 Hardy G. H. and Wright E. M., An Introduction to Theory of Numbers, (Sixth Edition) Oxford University Press.

PS02EMTH24: C Programming and Mathematical Algorithms I

Note: 50 Marks (35 marks for external examination and 15 Marks for internal examination) for theory and 50 Marks for practical on computers. External examination will be of two hours for theory and three hours for practical.

Unit I — Structure of a C program, the concept of function, preprocessors in C, include statement, function prototype error, comments in C, data types in C, integer family, float family, character family, type casting of variables, arithmetic and relational operators in C, input – output functions, I/O format string, precision of numbers, field width, assignment operators, Mathematical expressions, logical expressions, precedence and associativity of operators, standard library functions, define statement, common programming errors. Arrays in C.

Unit II — Branches in C: if, if-else, else-if statements, goto statement, switch statement. Loops: while, do–while, for, break and continue statements, nesting of loops. Statement function definitions. Structures and unions: declaration of structures, accessing structure members, structure initialization, nested structure, array of structures, structure assignment, structure as a function arguments, unions. typedef declaration.

Unit III & IV — (List of practical to be performed on computer :)

Practical No.	Purpose of the program
1	Conversion of units: like mile to km. To convert Cartesian coordinates to polar coordinates and vice versa, to convert degree to radian and vice versa, to find simple interest.
2	To interchange the content of two variables, to find maximum of given 4 numbers, To check given no is odd or even, to check given year is a leap year or not, to find real roots of a quadratic equation, to find all roots of a quadratic equation, to prepare the result of a student.
3	To find n!, to find a^n, to print character table with their ASCII values, To print multiplication table, to find average height of male and female students, to find numbers of students getting first class second class, pass class and fail.
4	To print numbers for 1 to n such that each line contains m numbers, to check whether given number is prime or not, to check whether given number is perfect or not, to print first k prime numbers, to print all prime numbers $\leq$ k, to print first k perfect numbers, to print all perfect numbers $\leq$ k, to print Floyd's triangle.
5	To find sum of digits of a number, to check whether a given number is Armstrong or not (Sum of cubes of digits = the number e,g, $3^3 + 7^3 + 1^3 = 371$), to print a given number in to reverse order of its digits, to check whether given number is palindrome or not, to list first k palindrome numbers, to list all palindrome numbers which are $\leq$ k, to find the factors of a natural number.
6	To find the GCD of two numbers, to find the LCM of two numbers, to find $\sigma(n)$, to find $\tau(n)$, to find $\varphi(n)$, to find $\mu(n)$ (where $\sigma(n)$ is the sum of positive divisors of n < n, $\tau(n)$ is number of positive divisors of

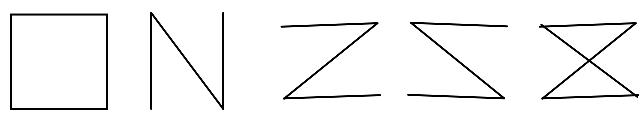
$n < n$, $\varphi(n)$ is the number of relatively prime numbers to n,
$\mu(n) = 1$ if $n = 1$
$\mu(n) = 0$ if $p^2 | n$ for some prime p
$\mu(n) = (-1)^r$, in $n = p_1 p_2 \ldots p_r$, where $p_1, p_2, \ldots, p_r$ are distinct prime numbers.)

7 To find e^x, to find sin(x), to find cos(x), to find sinh(x), to find cosh(x). (Terminate the programs after n terms. Terminate the programs if error is too small)

8 To write programs to count the number of students getting distinction, first class, second class, pass class and fail by using switch statement. Write programs to prepare a frequency distribution table, to prepare a menu generation.

9 To compute approximate solutions of the equation $f(x) = 0$ by using bisection method, regula falsi method, modified regula falsi method, secant method, Newton's method.

10 Write programs to generate figures like (by using a character):



Reference Books

1 Mahapatra P. B., Thinking in C Including object orientated programming with C++, Wheeler Publishing, New Delhi

2 Kernighan B. W. and Ritchie D. M., The C programming Language, Prentice Hall of India Pvt. Ltd., 1990.

3 Rajaraman V., Computer Programming in C, Prentice Hall of India Pvt. Ltd., 1995.