SARDAR PATEL UNIVERSITY VALLABH VIDYANAGAR



SYLLABUS EFFECTIVE FROM: 2018-19 MSC (Mathematics) Semester IV

There will be six courses, each of 3 credit. The session work for each course will comprise of 3 formal lectures per week, each of 1 hour duration, and 1 seminar and a problem session of 1 hour and 45 minute duration in each course per week. There will be a 1 credit course for comprehensive viva and Mathematics presentation. Thus a student will be provided 30 hours actual teaching per week; and he/she will be required to earn 25 credits during the semester. Each course will have a weighting of 100 marks (70 marks for University examination + 30 marks for inter Assessment. Internal Assessment will comprise of 1 internal test for 20 marks, a seminar of 5 marks and 5 marks for quiz.) Each student will take 6 courses in consultation and with approval of the department. There will be 5 core courses and 1 elective course to be taken by a student.

Viva: There will be a viva-voce examination of 50 marks at the end of each semester covering all the courses offered during the semester.

List of courses

Core Courses

PS04CMTH21: Complex Analysis II PS04CMTH22: Mathematical Methods II PS04CMTH23: Comprehensive Viva

Elective Courses

PS04EMTH31: Banach Algebras

PS04EMTH32: Computer Programming and Software

PS04EMTH33: Financial Mathematics I

PS04EMTH34: Financial Mathematics II

PS04EMTH35: Graph Theory I

PS04EMTH36: Graph Theory II

PS04EMTH37: Group Theory

PS04EMTH38: Mathematical Probability Theory

PS04EMTH39: Number Theory and Cryptography

PS04EMTH40: Operations Research

PS04EMTH41: Operator Theory

PS04EMTH42: Problems and Exercises in Mathematics II

PS04EMTH43: Relativity I PS04EMTH44: Relativity II PS04EMTH45: Topology II

PS04CMTH21: Complex Analysis II

- Unit I Riemann Stieltjes integral: a function of bounded variation on [a, b], its total variation, rectifiable curve, smooth curve, polygonal path, integral of a continuous function on [a, b] with respect to a function of bounded variation and its properties, integral of continuous function defined on $\{\gamma\}$ with respect to γ and its properties, zeros of an analytic function, multiplicity of zero of an analytic function, the index of a closed curve and its properties.
- Unit II Cauchy's Theorem and Integral Formula, Morera's Theorem, Counting zero principle and open mapping theorem, Classification of singularities namely removable singularity, pole and essential singularity, order of a pole.
- Unit III Argument Principle, Rouche's theorem, Schwarz's lemma and applications, the space of continuous functions $C(G, \Omega)$, the topology on $C(G, \Omega)$, normal family in $C(G, \Omega)$, equicontinuity of a family in $C(G, \Omega)$, Arzela Ascoli theorem in $C(G, \Omega)$.
- Unit IV The space H(G) of analytic function, locally bounded family in H(G), Hurwitz's theorem, Montel's theorem, infinite product, convergence and absolute convergence of infinite product, convergence of infinite product of elements in H(G), elementary factors and its properties, The Weierstrass Factorization Theorem, factorization of sin, cos, sinh and cosh, Walli's formula.

Textbook

1 J. B. Conway, Functions of One Complex Variable, 2nd Edition, Narosa, New Delhi, 1978.

Chapter IV: section 1, section 3 (except 3.3, 3.4, 3.5 and 3.11), section 4, section 5, section 6: 6.16 and 6.17 only, section 7

Chapter V: section 1 (except 1.8 to 1.20), section 3

Chapter VI: section 1(statements only), section 2

Chapter VII: section 1, section 2, section 5 (except 5.15 to 5.20), section 6

- 1 W. Rudin, Real and Complex Analysis, McGraw Hill, 1967.
- 2 R. Narasimhan and Y. Nievergelt, Complex Analysis in One Variable, Birkhauser, 2001.
- 3 Theodore W. Gamelin, Complex Analysis, Springer-Verlag New York, 2001

PS04CMTH22: Mathematical Method II

- Unit I Functional, Euler's equation, other forms of Euler's equation, some special forms of Euler's equation, geodesics, Isoperimetric problems, several dependent variables, functional involving higher order derivatives.
- Unit II Integral equations, types of integral equations, conversion of differential equation into an integral equation and vice versa, solution of integral equation, Integral equations of convolution type, Abel's integral equations, Integro-differential equation.
- Unit III Compact operators, some properties of compact operators, compact operators on Banach spaces C[a, b] and $L^2[a, b]$, Fredholm integral equations, Seperable/Degenerate kernel, Symmetric/Hermitian kernel, Iterated kernel, Resolvent kernel, Fredholm alternative theorem, solutions of Fredholm integral equations for separable kernels.
- Unit IV Bessel's equation, Chebyshev's equation, Laguerre's equation, Hermite equation, Laguerre equation, Legendre equation, Sturm-Liouville equations, Conversion of various types of differential equations into one of the above differential equations, Solution of Sturm-Liouville equation.

Textbook

1 B. S. Grewal, Higher Engineering Mathematics, KhannaPubls, 3rd Edition, Delhi. Chapter-16 (Section 16.19), Chapter-30 (Sections 30.1 to 30.8) and Chapter-31 (Sections 31.1 to 31.3, 31.5 to 31.10).

- 1 A. S. Gupta, calculus of variations with applications, Prentice-Hall of India, New Delhi, 1999.
- 2 B. V. Limaye, Functional analysis, 2nd Edition, New Delhi, 1996.
- 3 A. L. Rabenstein, Introduction to Ordinary Differential Equations, Academic Press, London, 1972.

PS04EMTH31: Banach Algebras

- Unit I Definition and Examples of Banach algebras, C(X), A(D), BL(X), $L^1(G)$ Regular and singular elements, topological divisors of zero, the spectrum, the Gel'fand Mazur's theorem.
- Unit II Spectral mapping theorem for polynomials, spectral radius formula, radical and semisimplicity, the Gel'fand mapping.
- Unit III Applications of the spectral radius formula, involutions in Banach algebras, the Stone Weierstrass theorem, the Gel'fand-Neumark theorem for commutative (abstract) C^* -algebras.
- Unit IV Ideals in C(X) and Banach-Stone theorem, the commutative C^* -algebra of bounded linear operators on a Hilbert space, Computation of Gel'fand space of some Banach algebras.

Textbook

1 Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.

Sections: 36 (Theorem A and Theorem B only), 64, 65, 66, 67, 68, 69, 68, 69, 70, 71, 72, 73, 74, 75, 76

- 1 E. Kaniuth, A Course in Commutative Banach Algebras, Springer, New York, 2009.
- 2 R. Larsen, Banach Algebras, Marcell-Dekker, 1973.
- 3 Banach Algebras and Automatic Continuity, London Mathematical Society, Monographs, 2001.

PS04EMTH32: Computer Programming and Software

Note: 50 Marks (35 marks for external examination and 15 marks for the internal examination) for theory and 50 Marks for practical on computers. External examination will be of two hours for theory and three hours for practical.

- Unit I The Basics: Literal constants, numbers, strings, variables, identifier naming, data types, objects, logical and physical lines, indentation. Operators, operator precedence, expressions. Control flow: the if statement, the while statement, the for loop, the break statement, the continue statement.
- Unit II Functions: Defining a function, local variables, default argument values, keyword arguments, the return statement, DocStrings. Modules: using the sys module, the from import statement, creating modules, the dir() function.
- Unit III List of computer practical.

& IV

Practical Purpose of the program

- 11. To find the minimum/maximum of a given list of numbers.
- 12. To check whether a given number is odd or even. To check whether a given year is a leap year or not.
- 13. To find the real roots of a quadratic equation.
- 14. To compute n!, aⁿ, sum and average of a list of numbers. To prepare the result of a student.
- 15. Primality lists: To check whether a given number is prime or not, to list all the prime numbers within a given range, to factorize a number.
- 16. Manipulation of numbers: to check whether a given number is perfect or not, to check whether a given number is palindrome or not, to compute the sum of digits of a given number, to compute the sum of squares of the digits, to print a given number in reverse order of its digits.
- 17. To compute GCD and LCM of two numbers, to evaluate the functions $\sigma(n)$, $\tau(n)$, $\phi(n)$, $\mu(n)$ for a given positive integer n.
- 18. To generate Fibonacci sequence and Lucas sequences; to compute the sum of the series and hence evaluate e^x , sin(x), cos(x), tan(x), sinh(x), cosh(x) (terminate the program after n terms of the series or terminate the program at the desired level of accuracy).
- 19. Basics of Scilab 1: Sum of matrices, determinant of a matrix, product of matrices, inverse of a matrix, row reduced echelon form.
- 20. Basics of Scilab 2: Plotting Cartesian, polar and parametric curves, commands for plotting functions.

Textbook

1 Swaroop C. H., A byte of Python.

Chapters: 4, 5, 6, 7, 8.

- 1 James Payne, Beginning Python: Using Python 2.6 and Python 3, Wiley India, 2010.
- 2 AmitSaha, Doing Math with Python, No Starch Press (2015).
- 3 SCILAB- A Free software to MATLAB by Er. HemaRamachandran and Dr. Achuthsankar S. Nair., S. Chand and Company Ltd. (2008).

PS04EMTH33: Financial Mathematics I

- Unit I Types of financial derivatives, Exchange Traded (ET) markets, Over The Counter (OTC) markets, Forward contracts, Futures contracts, Options, Types of traders: Hedgers, Speculators, Arbitrageurs, Uses of derivatives.
- Unit II Background of futures contracts, Specifications of futures contracts, Convergence of futures price, Daily settlement and margins, Delivery of futures contracts, Distinctions and comparisons between forward and futures contracts, Types of interest rates: continuously compounding, treasury rate, LIBOR, Repo rate, n-year zero rate, Forward rate.
- Unit III Short selling, Forward price for an investment asset, Valuing forward contracts, Comparisons of forward and futures prices, Futures and forward contracts on currencies, Futures contracts on commodities.
- Unit IV Types of options, Option positions, Underlying assets, Factors affecting option prices, Upper and lower bounds for option prices, Put-call parity, Early exercise, Effect of dividends.

Textbook

1 J. C. Hull and S. Basu, Options, Futures and Other Derivatives, 7th edition, Pearson Prentice Hall, 2011.

Chapter 1: Sec 1.1 - 1.9

Chapter 2: Sec 2.1 - 2.4, 2.6

Chapter 4: Sec 4.1 - 4.3, 4.6

Chapter 5: Sec 5.2, 5.4, 5.7, 5.8, 5.10, 5.11

Chapter 8: Sec 8.1 - 8.3

Chapter 9: Sec 9.1 - 9.7

2 S. L. Gupta, Financial Derivatives: Theory, Concepts and Problems, Prentice Hall of India, 2005.

Chapter 1: Sec 1.4, 1.7

Chapter 3: Sec 3.5

- 1 P. Wilmott, S. Howison and J. Dewynne, The mathematics of financial derivatives, Cambridge Uni. Press, 1995.
- 2 S. M. Ross, An elementary introduction to mathematical finance, Cambridge Uni. Press, 3rd edition, 2011.
- 3 S. N. Neftci, An introduction to the mathematics of financial derivatives, Academic Press, 2nd edition, 2000.

PS04EMTH34: Financial Mathematics II

- Unit I One step binomial model, Risk-Neutral valuation, Two step binomial trees, Markov property, Wiener process, Generalized Wiener process, Simple model for stock price, Ito process, Ito's lemma.
- Unit II Log normal property of stock prices, Distribution of the rate of return, Derivation of Black-Schole-Merton (BSM) differential equation, Derivation of BSM formulas for European options through probabilistic approach.
- Unit III Derivation of Greek letters: Delta, Theta, Gamma, Vega, Rho, Delta hedging, BSM formulas on an asset paying constant dividend, BSM formulas for currency options.
- Unit IV Put-call parity for European futures options, Bounds for futures options, Valuation of futures options using binomial trees, BSM formulas for futures options.

Textbook

1 J. C. Hull and S. Basu, Options, Futures and Other Derivatives, 7th edition, Pearson Prentice Hall, 2011.

Chapter 11: Sec 11.1 – 11.4

Chapter 12: Sec 12.1 – 12.5 (except Monte Carlo simulation)

Chapter 13: Sec 13.1, 13.2, 13.5, 13.6, 13.8

Chapter 15: Sec 15.3, 15.5

Chapter 16: Sec 16.3 – 16.6, 16.8

Chapter 17: Sec 17.4 – 17.9

 S. M. Ross, An elementary introduction to mathematical finance, Cambridge Uni. Press, 3rd edition, 2011.
 Chapter 7: Sec 7.5 (Lemma 7.5.1 - 7.5.3 & Theorem 7.5.1 only)

- 1 P. Wilmott, S. Howison and J. Dewynne, The mathematics of financial derivatives, Cambridge Uni. Press, 1995.
- 2 S. L. Gupta, Financial Derivatives: Theory, Concepts and Problems, Prentice Hall of India, 2005.

PS04EMTH35: Graph Theory I

- Unit I Review of basic facts about graphs: connected graph, distance and diameter, tree, Euler graph, fundamental circuits, matrix representation of graphs, isomorphic graphs.
 Directed Graphs: Definitions and examples, vertex degrees, some special types of digraphs, directed path and connectedness, Euler digraphs.
- Unit II Trees with directed edges, spanning out-tree, spanning in-tree and their relation with Euler digraph, Incidence matrix, Circuit matrix and Adjacency matrix of digraphs, Fundamental circuits and fundamental circuit matrix in digraphs.
- Unit III Chromatic number, chromatic partitioning, Uniquely colorable graphs, Chromatic polynomial, Four-color Problem. Hamiltonian cycles: necessary conditions, sufficient conditions.
- Unit IV Matching and Covers: maximum matching, Hall's matching condition, min-max theorems, independent sets, vertex cover, edge cover.

Textbook

- NarsinghDeo, Graph Theory with applications to Engg. and Computer Science, Prentice-Hall of India Pvt. Ltd., New Delhi, 1999.
 Chapter 9: Sections 9.1 to 9.9 (omit 9.3, Kirchhoff matrix from 9.9), Chapter 8: Sections 8.1 to 8.3(omit dominating sets, 8.6)
- 2 Douglas B. West, Introduction to Graph Theory, Pearson Education, Inc. 2002. Chapter 3: Section 3.1(up to 3.1.24), Chapter 7: Section 7.2(up to 7.2.8) & 7.2.19

- 1 John Clark and Holton D.A., A first look at graph theory, Allied Publishing Ltd., 1991.
- 2 Robin J. Wilson, Introduction to graph theory, Pearson Education Asia Pvt. Ltd., 2000.

PS04EMTH36: Graph Theory II

- Unit I Eigen values of graphs: Definition & basic properties, examples, eigen values of bipartite graphs, eigen values and graph parameters Diameter, $\Delta(G)$ and $\delta(G)$, chromatic number, regularity & connectedness.
- Unit II Network: Flows and cuts, maximal flow, Min-max theorem. Ramsey theory: The Pigeonhole principle & its applications, Ramsey numberdefinition, graph theoretic representation for r = 2, Ramsey's theorem (Equivalent statements), lower and upper bound for Ramsey number.
- Unit III Enumeration of Trees: Cayley's formula, degree sequence of graphs. Spanning Trees in graphs: Contraction by edge, matrix-tree theorem. Decomposition and graceful labeling.
- Unit IV Minimum spanning trees: Kruskal's algorithm, Prim's algorithm. Shortest Path Problems: Breadth First Search algorithm, Back-tracking algorithm, Dijkstra's algorithm for weighted graphs.

Textbook

1 Douglas B. West: Introduction to Graph Theory, Pearson Education Pvt. Ltd., India, 2001.

Sections: 2.2 – 2.2.1 to 2.2.19, 8.3 – 8.3.1 to 8.3.6, 8.3.9 to 8.3.11, 8.6 – 8.6.1 to 8.6.9 [omit 8.6.4 to 8.6.6], 8.6.16 to 8.6.27 [omit 8.6.20 to 8.6.22]

2 John Clark and D. A. Holton: A First look at Graph Theory, Allied Publishing Ltd., 1991.

Sections 2.4 & 2.5, Section 8.1

- 1 NarsinghDeo: Graph Theory with applications to Engg. And Computer Science, Prentice-Hall of India Pvt. Ltd., New Delhi, 1999.
- 2 Russell Merris, Graph Theory, Wiley-Interscience, John Wiley & Sons, Inc., 2001.

PS04EMTH37: Group Theory

- Unit I Definition of a group, some examples of groups, some preliminary lemmas, subgroups, two equivalence relations $a \equiv bmodH$ if $ab^{-1} \in H$ and $a \sim modH$ if $a^{-1}b \in H$, Lagrange's theorem, Euler's theorem, Fermat's theorem, counting principle, the condition for *HK* to be a subgroup and determination of o(HK), normal subgroups, and quotient groups, characterizations of normal subgroups, homomorphism, isomorphism, first isomorphism theorem, simple group, Cauchy's theorem for abelian groups.
- Unit II Automorphism, inner automorphism, Cayley's theorem and its applications, permutation groups, permutation as a product of disjoint cycles and transpositions, even and odd permutations, alternating group, another counting principle, conjugate classes, class equation and its applications, Cauchy's theorem (general case), number of conjugate classes in S_n .
- Unit III Sylow's theorem, first proof, definition of p-Sylow subgroup, second proof of Sylow's theorem, existence of p-Sylow subgroup in S_{p^k} , double cosets and its order, existence of p-Sylow subgroup in subgroups, second part of Sylow's theorem, number of p-Sylow subgroups in a group, third part of Sylow's theorem, examples based on Sylow's theorems.
- Unit IV Direct products, external direct product and internal direct product, properties of internal direct product, finite abelian groups as direct product of cyclic groups, invariants of an abelian group of order p^n for prime p, the subgroup G(s) of an abelian group G, for an integer $so(G(p^m))$ for a prime p, uniqueness of invariants, number of non-isomorphic abelian groups of order $p_1^{\alpha_1} \cdots p_r^{\alpha_r}$, where p_i are distinct primes and $\alpha_i > 0$.

Textbook

1 Herstein, I. N., Topics in Algebra, (Second Edition), Wiley Eastern Ltd., New Delhi, 1975.

Chapter 2: § 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7 (except Application 2 and its Corollary, Lemma 2.7.5 and Theorem 2.7.2), 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.14

- 1 Fraleigh J. B., A First Course in Abstract Algebra (Third Edition), Narosa, 1983.
- 2 Gallian J. A., Contemporary Abstract Algebra (Fourth Edition), Narosa, 2008.

PS04EMTH38: Mathematical Probability Theory

- Unit I Random variables, Vector random variables, Limits of random variables, General probability space, Induced probability space, Distribution function of random variable, Jordan Decomposition theorem.
- Unit II Distribution function of vector random variables, Distribution function of dense subset of R, Expectation, Properties of expectation, Expectation of complex random variables, C_r - Inequality, Holder's Inequality, Minkowski Inequality, Jensen's Inequality, Chebyshev's Inequality.
- Unit III Convergence in probability, Weak law of large numbers, Convergence almost surely, Strong law of large numbers, Convergence in distribution, Convergence in r^t mean, Monotone convergence theorem, Fatou's theorem, Dominated convergence theorem.
- Unit IV Characteristic function, Properties of characteristic function, Inversion formula, Helly's theorems, Helly-Bray theorem, Levy's theorem (continuity theorem), Kolmogorov's Inequality, Central limit theorem (Lindeberg-Levy's theorem).

Textbook

- B. R. Bhat, Modern Probability Theory: An Introductory Textbook, New Age International Publishers, 4th edition, 2014.
 Unit 2: Sec 2.2(a)(b), 2.3(b)(c)
 Unit 3: Sec 3.4(a), 3.5(a)
 Unit 4: Sec 4.1(a), 4.2(a)(b), 4.3(a), 4.4(a)
 Unit 5: Sec 5.1(a)(b)(c), 5.2(a)(b)(c), 5.3(c)(d)
 Unit 6: Sec 6.1(a)(b)(c)(d), 6.2(a), 6.3(a)(b), 6.4(a)(b), 6.5(a)(b)(c)
 Unit 7: Sec 7.1(a)(b)(c), 7.2(a)(b), 7.3(a) (Theorem 7.2 only), 7.3(b)
- A. K. Basu, Measure Theory and Probability, Prentice Hall of India, 2nd edition, 2015. Chapter 6: Sec 6.1 (Lemma 6.1 only)
 Chapter 8: Sec 8.1, 8.3 (Theorem 8.3 & Theorem 8.4 only)
 Chapter 9: Sec 9.6 (Theorems 9.12, 9.13, 9.14, 9.15 only)
 Chapter 10: Sec 10.1

- 1 P. Wilmott, S. Howison and J. Dewynne, The mathematics of financial derivatives, Cambridge Uni. Press, 1995.
- 2 S. M. Ross, An elementary introduction to mathematical finance, Cambridge Uni. Press, 3rd edition, 2011.
- 3 S. N. Neftci, An introduction to the mathematics of financial derivatives, Academic Press, 2nd edition, 2000.

PS04EMTH39: Number Theory and Cryptography

- Unit I Number Theory and Discrete Logarithm Problem: Simple substitution ciphers (except cryptanalysis), divisibility and GCD, modular arithmetic, prime numbers, unique factorization and finite fields, primitive roots in finite fields. The discrete logarithm problem.
- Unit II DLP based cryptosystems: The Diffie-Hellman key exchange, the ElGamal public key cryptosystem, difficulty of discrete log problem (DLP), a collision algorithm for the DLP, the Chinese remainder theorem, the Pohlig-Hellman algorithm.
- Unit III The RSA Algorithm: Euler's formula and roots modulo pq, the RSA public key cryptosystem, implementation and security issues, primality testing, Pollard's p-1 factorization algorithm.
- Unit IV Elliptic curve cryptography: Elliptic curves, elliptic curve over finite fields, the elliptic curve discrete logarithm problem, elliptic curve cryptography.

Textbook

 Hoffstein J., Pipher J., Silverman J. H., An Introduction to Mathematical Cryptography, Undergraduate Texts in Mathematics, Springer, New York, (2008).
 Sections: 1.1 (except 1.1.1), 1.2, 1.3, 1.4, 1.5, 1.7, 2.2; 2.3, 2.4, 2.6, 2.7, 2.8, 2.9; 3.1, 3.2, 3.3, 3.4, 3.5; 5.1, 5.2, 5.3, 5.4.

- 1 Douglas R. Stinson, Cryptograph: Theory and Practice, Second Edition, Chapman and Hall/CRC, (2005).
- 2 N. Koblitz, A Course in Number Theory and Cryptography, Springer (1994).
- 3 J. A. Buchmann, Introduction to Cryptography, Second Edition, Undergraduate Texts in Mathematics, Springer-Verlag, New York, (2004).

PS04EMTH40: Operations Research

- Unit I Modelling with Linear Programming: two-variable LP model, graphical LP solution, LP model in equation form, the simplex method, artificial starting solution, M-method, Two-Phase method, graphical sensitivity analysis.
- Unit II Special Cases in the Simplex Method: degeneracy, alternative optima, unbounded solution, infeasible solution, definition of dual problem, primal-dual relationships, economic interpretation of duality, dual simplex algorithm.
- Unit III Transportation Model and its Variants, definition of the transportation model, nontraditional transportation models, the transportation algorithm, determination of starting solution, iterative computations of the transportation algorithm, simplex method explanation of the method of multipliers, the Assignment model, the Hungarian method, Simplex explanation of the Hungarian method.
- Unit IV Classical Optimization Theory: unconstrained problems, necessary and sufficient conditions, constrained problems, equality constraints, inequality constraints Karush-Kuhn-Tucker (KKT) conditions.

Textbook

1 Hamdy A. Taha, Operations Research: An Introduction, Pearson, Ninth Edition (2012).

Sections: 2.1, 2.2, 3.1, 3.3, 3.4, 3.6 (3.6.1 only); 3.5, 4.1, 4.2, 4.3, 4.4 (4.4.1 only); 5.1, 5.2, 5.3, 5.4; 20.1 (20.1.1 only), 20.2.

- 1 KantiSwarup, Gupta P.K., and Man Mohan, Operations Research, (2004), S. Chand & sons.
- 2 G. Hadley, Linear Programming, Addition Wesley Publ. (1962).
- 3 F. S. Hillier and G. J. Lieberman, Introduction to Operations Research, Mcgraw-Hill Higher Education, (2001).

PS04EMTH41: Operator Theory

- Unit I Review of Hilbert space H, Orthogonal complement in H, Dual of H, Bounded operator, Existence of adjoint operator and its properties, Self-adjoint operator and its properties, Unitary operator and its properties, Fuglede-Putnam-Rosenblum theorem (i.e., Commutativity Theorem)
- Unit II Resolution of the identity E, the algebra $L^{\infty}(E)$, identifying $L^{\infty}(E)$ with a closed subalgebra of BL(H), Spectral theorem and its applications, Spectral decomposition.
- Unit III Symbolic calculus for normal operators and its applications on normal operators, Invariant subspace problem, Eigenvalue of normal operators, Positive operators and square roots, Polar decomposition and its uniqueness, Unitarily equivalent operators.
- Unit IV Hilbert-Schmidt operators and their properties, Multiplier operator T_f on l^2 , Classification of the operator T_f in terms of f, Trace class operators, Hilbert-Schmidt and trace class norms, relations between these two types of operators

Textbook

- 1 W. Rudin, Functional Analysis, Tata McGraw Hill Pub. Company, New Delhi, 1973. Chapter-12 (Sections 1 to 36)
- J. B. Conway, A Course in Operator Theory, Graduate Studies in Mathematics, Volume 21, American Mathematical Society, Rhode Island, 2000.
 Section-18

Reference Books

1 B. V. Limaye, Functional analysis, 2nd Edition, New Age International Limited, New Delhi, 1996.

PS03EMTH42: Problems and Exercises in Mathematics II

Students will be required to prepare him/herself for any five of the following courses at the level up to M.Sc. third semester of Sardar Patel University for problems and exercises. The regular teaching involves intensive problem sessions followed by problem assignments; and the examination would consist of problems only. Algebra, Topology, Real Analysis, Complex Analysis, Functional Analysis, Differential Geometry, Classical Mechanics, Linear Algebra, Differential Equations.

PS04EMTH43: Relativity I

- Unit I Historical background and postulates of special relativity, relativity of simultaneity, Michelson Morley experiment, Special Lorentz transformation, consequences of special Lorentz transformation, addition of velocities, special Gelien transformation.
- Unit II Aberration of light, Doppler Effect, space-time interval, four dimensional formulation, poincare structure of spacetime, Minkonski structure of spacetime
- Unit III Covariance four dimensional form, principle of covariance, proper time, four dimensional vectors (Displacement velocity), mass of moving particle, covariant form of Newtonian's law, momentum 4-vector, relativistic kinetic energy, equivalence of mass and energy.
- Unit IV Principle of equivalence, brief introduction of general relativity, definition of tensor, fundamental tensor of spacetime, Christoffel symbols, geodesic equation, gravity as geometric phenomena.

Textbook

- 1 Resnick, R., Introduction to Special Relativity, Wiley (2.1, 2.7, C)
- Banerji, S. and Banerjee, A. The Special Theory of Relativity, Prentice-Hall of India, Delhi, 2012

(2.1, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.4, 3.5, 3.6, 3.7)

³ Adler, R., Bazin, M. And Schiffer, M. Introduction to General Relativity, McGraw-Hill LTD. 1975.

(1.1, 1.2, 1.3, 1.4, 1.7, 1.8, 2.2, 2.3, 2.3, 4.3)

PS04EMTH44: Relativity II

- Unit I Space-time fundamental tensors, Christoffel symbols, Riemann tensor, Ricci tensor, Einstein tensors in general relativity. Energy-momentum tensor, criteria for gravitational field equations and Einstein's field equations.
- Unit II Metric for spherically symmetric space-times, Schwarzschild exterior solution, various forms of Schwarzschild solution. The general relativistic Kepler problem and crucial tests of GR, Kruskal coordinates and the black hole, Schwarzchild interior solution.
- Unit III Relativistic cosmology: observational background, cosmological postulates, static models of the universe. Features of Einstein universe and deSitter universe. Limitations of static models.
- Unit IV Cosmological principle, Weyl hypothesis, Robertson-Walker metric, Einstein's field equations for RW metric, Robertson-Walker Cosmological models, features of the RW models.

Textbook

1 Adler, R., Bazin, M. and Shiffer, M., Introduction to general relativity (Second Edition). McGraw Hill.

(6.1, 6.2, 6.3, 6.5, 6.6, 6.8)

- 2 ZafarAhsan, Tensors mathematics of differential geometry and general relativity. (2.2, 3.2, 3.4, 4.2, 4.3, 4.4, 4.5, 4.7, 4.8, 4.9, 6.4)
- 3 Narlikar, J.V., An Introduction to Cosmology, Cambridge University Press, Cambridge. 2002.

(3.1, 3.2 3.3, 3.4, 3.5, 3.6, 4.1, 4.2, 4.3, 4.4)

PS04EMTH45: Topology II

- Unit I Neighbourhoods, neighbourhood base at a point, Product spaces and the weak topology Inadequacy of sequences, directed set, net, convergence and clustering of a net, characterization of closure and continuity using net, subnet, ultranet.
- Unit II Filter, filter base, convergence and clustering of a filter, finer filter, ultra filter, free and fixed filter, characterization of closure and continuity using filter.
- Unit III Filter generated by a net, a net based on a filter, and their convergence. Characterization of compact spaces using nets and filters, Tychonoff Theorem.
- Unit IV Homotopy of functions from one topological space to another, path homotopy, product of two paths and its algebraic structure, loop, Fundamental group relative to the base point, isomorphism of fundamental groups, simply connected space, homomorphism induced by a continuous map.

Textbook

- 1 Willards, S., General Topology, Dover Publication, New York, 1970. Sections: 4, 8, 10, 11, 12 17 (only 17.4 and 17.8)
- 2 Munkres, J., Topology: A First Course, 2/e, Prentice Hall of India Pvt. Ltd. New Delhi, 2003. Sections: 51, 52

Reference Books

1 Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.