**SARDAR PATEL UNIVERSITY**
**VALLABH VIDYANAGAR**

**SYLLABUS EFFECTIVE FROM: 2017-18**
**Syllabus for M.Sc. (Mathematics)**
**Semester I**

There will be six courses, each of 4 credits. The session work for each course will comprise of four lectures each of one hour duration and if needed, based on the strength of the class, one hour per week for seminar. For a stipulated period of certain weeks, decided by the department/centre for each course separately, one hour out of the four allotted hours, may be reallocated to seminars. There will be a 1 credit course for comprehensive viva and Mathematics presentation for which students may seek guidance from concerned teachers. Thus a student will be provided 30 hours actual teaching per week; and he/she will be required to earn 25 credits during the semester. Each course will have a weighting of 100 marks (70 marks for University examination + 30 marks for inter assessment. Internal assessment will comprise of 1 internal test of 20 marks, a seminar of 5 marks and quizzes of 5 marks). Each student will take 6 courses in consultation and with the approval of the department. There will be 5 core courses and 1 elective course to be taken by a student.

**Viva:** There will be a viva-voce examination of 50 marks at the end of each semester covering all the courses offered during the semester.

## List of courses

# Core Courses

PS01CMTH21: Complex Analysis I
PS01CMTH22: Topology I
PS01CMTH23: Functions of Several Real Variables
PS01CMTH24: Linear Algebra
PS01CMTH25: Methods of Differential Equations
PS01CMTH26: Comprehensive Viva

# Elective Courses

PS01EMTH21: Graph Theory I
PS01EMTH22: Mathematical Classical Mechanics
PS01EMTH23: Number Theory
PS01EMTH24: C Programming and Mathematical algorithms I

## PS01CMTH21: Complex Analysis I

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|---|---|
| Unit I | Metric on C, Polar representation and nth-roots of a complex number, limit and continuity of a complex function, derivative of a complex function, Cauchy-Riemann equations |
| Unit II | Analytic functions, harmonic functions, power series, power series as an analytic function, elementary functions $\exp z$, $\cos z$, $\sin z$, $\cosh z$, $\sinh z$. |
| Unit III | Contours, contour integrals, anti-derivative, Cauchy's theorem, Cauchy's Integral Formula, Cauchy inequality, Liouville's theorem, Fundamental Theorem of Algebra, Morera's theorem, Cauchy-Goursat's theorem. Gauss mean value theorem, Principle of deformation of paths, maximum modulus principle. |
| Unit IV | Taylor's theorem, Laurent series, absolute and uniform convergence of power series. Classification of singularities, residues, residue theorem, residues at poles, evaluation of improper real integrals, definite integrals with sine and cosine function, Mobius transformation. |

**Textbook**

1 Churchil, R.V., Brown, J. and Verle, R., Complex Variables and Applications, McGraw-Hill Publ. Co., Eighth edition, 2009.

Chapter 1: Sections: 4, 5, 6, 7, 8, 9, Chapter 2: Sections: 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, Chapter 3: Sections: 29, 34, 35, Chapter 4: Sections: 37, 38, 39, 40, 41, 43, 44, 46, 50, 51, 52, 53, 54, Chapter 5: Sections: 57, 58, 59, 60, 61, 62, Chapter 6: Sections: 68, 69, 70, 72, 73, 75, 76, Chapter 7: Sections: 78, 79, 80, 81, 82, 85, Chapter 8: Sections: 93, 94

**Reference Books**

1 Conway J.B., Functions of One Complex Variable, (Second Edition), Narosa Publ. House, New Delhi, 1994.

2 Ponnusamy S., Foundations of Complex Analysis, Narosa Publ. House, New Delhi, 1995.

3 Choudhary B., The Elements of Complex Analysis, (Second Edition), Wiley Eastern Ltd., New Delhi, 1992.

## PS01CMTH22: Topology I

| | |
|---|---|
| Unit I | Topological spaces, basis, subbasis, the product topology on X × Y, the subspace topology, closed sets, closure and interior, limit points, Hausdorff spaces, convergent sequence, $T_1$-space. |
| Unit II | Continuous functions, homeomorphisms, constructing continuous functions, pasting lemma, the product topology, maps into product, metric topology, diameter and bounded sets, continuity in metric spaces, the sequence lemma, first countability axiom. |
| Unit III | Connected spaces, connected subspaces of the real line, compact spaces, Heine-Borel theorem for real line, second countable spaces, separable spaces, regular spaces. |
| Unit IV | Normal spaces, Urysohn's Lemma (statement only), Tietze's Extension Theorem, (statement only) complete metric spaces, Cantor's intersection theorem, Baire's category theorem for complete metric spaces. |

**Textbooks**

1 Munkres, J., Topology: A First Course, (Second Edition), Prentice Hall of India Pvt. Ltd. New Delhi, 2003.

§ 12, 13, 15, 16, 17 (Except 17.11), 18, 20 (Only up to 20.3), 21, 23, 24 (Only up to 24.2), 26 (Except 26.7, 26.8), 30 (Except Lindelof space), 31 (except results concerning arbitrary product), 32 (up to 32.3), 33 (only 33.1).

2 Simmons G.F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.

§ 12, 21 (only Theorem G)

**Reference Books**

1 Willard S., General Topology, Dover Publication, 2004.

2 Kelley J., General Topology, Graduate Texts in Mathematics, Springer-Verlag, 1975.

# PS01CMTH23: Functions of Several Real Variables

Unit I Euclidean Space $R^n$, Euclidean norm and inner product on $R^n$, their basic properties and relation among them, open and closed balls, limits and continuity of functions $f : R^n \to R^m$, examples and basic results on limits and continuity, result on composition of continuous functions, oscillation of a bounded function $f : R^n \to R$, relation between continuity and oscillation, linear transformation (LT) from $R^n$ into $R^m$, norm of an LT, characterization of the dual space of $R^n$, norm preserving LT, inner product preserving LT, angle preserving LT, relation among these three properties for LT, relation between LT and matrices, examples and counter examples on all these concepts.

Unit II Two definitions of derivative of a function $f : R \to R$ (one is classical and the other one is in terms of LT) and their equivalence, some examples of derivatives in form of LT, total derivative $Df(a)$ of a function $f : R^n \to R^m$, uniqueness of derivative, examples of differentiable functions and non-differentiable functions, Jacobian matrix $f'(a)$, chain rule, basic theorems on differentiability, examples and counter examples on all these concepts.

Unit III Partial derivatives, second order partial derivatives (only definition and some examples), relation between Jacobian matrix and partial derivatives, relation between partial derivative and total derivative, continuously differentiable function and its relation with derivatives, directional derivative, its basic properties, its relation with derivative, partial derivative, and continuity, examples and counter examples on all these concepts.

Unit IV Tensor algebra on finite dimensional vector space V, basic results on k-tensors, dimension of $T^k(V)$ - the space of all k-tensors on V, examples of dual basis of $(R^n)^*$, dual function $f_k^*$ of a linear function $f : V \to W$, inner product on V, relation between inner products on V and on $R^n$, alternating k-tensor, Alt(T) of a k-tensor T, results on alternating tensors and Alt(T), wedge product, dimension of $\Lambda^k(V)$ – the space of all alternating k-tensors, fields and k-forms, basic results on fields and k-forms, examples and counter examples on all these concepts.

**Textbooks**

1 Spivak M., Calculus on Manifolds, W.E. Benjamin Inc., 1965.

Chapter-1, Chapter-2 (up to Theorem 2.8), Chapter-4 (up to Theorem 4.8)

2 Rudin W., Principles of Mathematical Analysis, (Third Edition), Tata McGraw-Hill Publ., New Delhi, 1983.

Chapter-9 (up to Theorem 9.21)

**Reference Book**

1 Ghorpade S. R. and Limaye B. V., A Course in Multivariable Calculus and Analysis, Springer, 2010.

## PS01CMTH24: Linear Algebra

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|---|---|
| Unit I | Quick review of vector spaces, examples of sequence and function spaces. Linear spans; linear dependence/independence and basis, examples of finite dimensional and infinite dimensional vector spaces, quotient space and its dimension, dual space, dual basis, dimension of the annihilator. Solution of the system of simultaneous linear homogeneous equations. |
| Unit II | Definitions and examples of algebra, algebra analog of Cayley theorem, minimal polynomial of a linear transformation, rank of a linear transformation, characteristic roots, characteristic vectors and results related to characteristic vectors, matrix associated with a linear transformation on finite dimensional vector space, isomorphism between the space of linear transformations and the space of matrices, similarity of matrices and similarity of linear transformations. |
| Unit III | Relation of the minimal polynomials of a linear transformation and its induced linear transformation on a quotient space, triangular matrix associated to a linear transformation, nilpotent linear transformation, canonical matrix associated to a nilpotent linear transformation, existence and uniqueness of invariants of a nilpotent linear transformation. Jordan form of a linear transformation. |
| Unit IV | Trace and its applications, Jacobson's lemma, transpose of a matrix. Definition of the determinant of a matrix, determinant of a triangular matrix, a matrix with equal rows, a product of matrices, application of determinant: regularity of a matrix, Cramer's rule to solve system of simultaneous non-homogeneous linear equations, quadratic forms: diagonalization of a symmetric matrix, symmetric matrix associated to a quadratic form, classification of quadrics. |

**Textbooks**

1 Herstein I. N., Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
Sections: 4.1, 4.2, 4.3, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.8, 6.9

2 Kwak J. H., Hong S., Linear Algebra, (Second Edition), Birkhauser, 2004.
Sections: 8.1, 8.2

**Reference Books**

1 Kumaresan S., Linear Algebra: A Geometric Approach, Prentice Hall of India, 2000.
2 Simmons G. F., Introduction to Topology and Modern Analysis, McGraw-Hill Co., Tokyo, 1963.
3 Helson H., Linear Algebra, (Second Edition), Hindustan Book Agency, TRIM-4, 1994.
4 Ramachandra Rao A. and Bhimasankaram P., Linear Algebra (Second Edition), Hindustan Book Agency, TRIM-19, 2000.

## PS01CMTH25: Methods of Differential Equations

| | |
|---|---|
| Unit I | Power series, real valued analytic function, second order linear differential equation: classification of singularities, series solution: near ordinary point, point at infinity, regular singular point using Frobenius method. |
| Unit II | A quick review of gamma function, Bessel's differential function, Bessel's function of first kind and its properties, Fourier-Bessel expansion theorem (statement only) and examples, Legendre's differential equation, Legendre polynomial and its properties, Rodrigue's formula, Fourier-Legendre's expansion theorem (statement only) and examples. |
| Unit III | Picard's method of successive approximations, Gauss's hypergeometric differential equation, Gauss's hypergeometric function and its properties, Gauss formula, Vandermondes formula, Kummer's formula, relation of Gauss's hypergeometric function and Legendre's polynomials. |
| Unit IV | Origin of first order partial differential equation, solution of first order partial differential equation using Lagrange's method, Pfaffian differential equation: homogeneous method, Natani's method for finding solution. |

**Textbooks**

1 Simmons G. F., Differential Equations with Applications and Historical Notes, (Second Edition), McGraw-Hill International Editions.
Chapter 5: Sec 26, 28 (Theorem A Statement only), 29, 30 (Theorem A Statement only), 32 (Except all appendices), Chapter 8: Sec 44, 45 (except least square approximation), 46, 47 (except Appendices A,B), Chapter13: Sec 68

2 Sneddon I. N., Elements of Partial Differential Equations, McGraw-Hill Publ. Co., 1957.
Chapter 1: Sec 5 (except Theorem 6), Sec 6(a)(b)(c)(d), Chapter 2: Sec 1, 2, 4 (Theorem 2 statement only)(except Theorem 3)

3 Raisinghania M. D., Advanced Differential Equations, (Sixth Revised Edition), S. Chand, 2013.
Part 1 Chapter 14: Sec 14.1 to 14.12 (except 14.3), 14.20.

**Reference Books**

1 Amarnath T., Elementary Course in Partial Differential Equations, Narosa Publ. House, New Delhi, 1997.

2 Rabenstein A. L., Introduction to Ordinary Differential Equations, Academic Press.

3 Grewal, B.S. and Grewal, J.S., Higher Engineering Mathematics, (36th Edition), Khanna Publ., New Delhi, 2000.

4 Somasundaram, D., Ordinary Differential Equations: A First Course, Narosa Publ. House, New Delhi, 2002.

## PS01EMTH21: Graph Theory I

| | |
|---|---|
| Unit I | Review of basic facts about graphs: connected graph, distance and diameter, tree, Euler graph, fundamental circuits, matrix representation of graphs, isomorphic graphs.<br>Directed Graphs: Definitions and examples, vertex degrees, some special types of digraphs, directed path and connectedness, Euler digraphs. |
| Unit II | Trees with directed edges, spanning out-tree, spanning in-tree and their relation with Euler digraph, Incidence matrix, circuit matrix and adjacency matrix of digraphs, fundamental circuits and fundamental circuit matrix in digraphs. |
| Unit III | Chromatic number, chromatic partitioning, uniquely colorable graphs, chromatic polynomial, four-color problem. Hamiltonian cycles: necessary conditions, sufficient conditions. |
| Unit IV | Matching and covers: maximum matching, Hall's matching condition, min-max theorems, independent sets, vertex cover, edge cover. |

**Textbooks**

1 Deo Narsingh, Graph Theory with Applications to Engg. and Computer Science, Prentice-Hall of India Pvt. Ltd., New Delhi, 1999.
Chapter 9: Sections 9.1 to 9.9 (Except 9.3, Kirchhoff matrix from 9.9), Chapter 8: Sections 8.1 to 8.3(Except dominating sets, 8.6)

2 West Douglas B., Introduction to Graph Theory, Pearson Education, Inc. 2002.
Chapter 3: Section 3.1(up to 3.1.24), Chapter 7: Section 7.2 (up to 7.2.8) & 7.2.19

**Reference Books**

1 Clark J. and Holton D.A., A First Look at Graph Theory, Allied Publishing Ltd., 1991.

2 Robin J. Wilson, Introduction to Graph Theory, Pearson Education Asia Pvt. Ltd., 2000.

## PS01EMTH22: Mathematical Classical Mechanics

| | |
|---|---|
| Unit I | Constraints and their classification, principle of virtual work, de'Almbert's principle, various forms of Lagrange's equations of motion for holonomic systems, examples. |
| Unit II | Euler-Lagrange equations in various forms (statements only), Hamilton's variational principle, derivation of Lagrange's equation from Hamilton's variational principle, generalized momentum, cyclic coordinates, general conservation theorem, conservation of linear momentum and angular momentum in Lagrangian formalism and symmetry properties, energy function and conservation of total energy in Lagrangian formalism. |
| Unit III | Hamilton's canonical equation of motion, relation with Lagrange's equation, cyclic coordinate, Routhian procedure, variational principle approach to Hamilton's equation of motion, examples. |
| Unit IV | Canonical transformations, generating functions, symplectic condition, infintesimal canonical transformations, examples. Poisson bracket, Lagrange bracket, formal solution of equations of motion in terms of Poisson brackets, examples. |

**Textbook**

1 Goldstein, H., Poole, C. and Safko, J., Classical Mechanics, (Third Edition), Pearson Education, Inc., Indian Low Price Edition, 2002.
Articles: 1.3, 1.4, 1.5 and 1.6, 4.1 (understanding of constraints and generalized coordinates in a rigid body motion); 2.1, 2.2 (statements only), 2.3, 2.6 and 2.7; 8.1,8.2, and 8.5; 9.1,9.2, 9.4, 9.5 and 9.6.

**Reference Books**

1 Bhatia V. B., Classical Mechanics, Narosa Publishing House, 1997.
2 Sankara Rao K., Classical Mechanics, Prentice-Hall of India, 2005.

## PS01EMTH23: Number Theory

| | |
|---|---|
| Unit I | The division algorithm, the greatest common divisor, the Euclidean algorithm, the fundamental theorem of arithmetic, infinitude of prime numbers (Euclid's proof), basic properties of congruence, linear congruences and the Chinese remainder theorem. |
| Unit II | Fermat's little theorem, Wilson's theorem, the sum and number of divisors, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, some properties of the phi-function. |
| Unit III | Euler's criterion, Legendre's symbol: definition and its properties, evaluation of $(-1|p)$ and $(2|p)$, Gauss lemma, quadratic reciprocity. |
| Unit IV | Algebraic numbers, Algebraic integers, quadratic fields, units and primes in quadratic fields, unique factorization. |

**Textbooks**

1 Burton David M., Elementary Number Theory, (Seventh Edition) McGraw Hill Education.

2.2, 2.3, 2.4, 3.1, 3.2, 4.2, 4.4, 5.2 (except theorems 5.2, 5.3), 5.3, 6.1, 6.2, 7.2, 7.3, 7.4, 9.1, 9.2, 9.3

2 Nivan Ivan, Zuckermann H. S. and Montgomery H. L., An Introduction to the Theory of Numbers, (Fifth Edition) John Wiley & Sons Inc.

9.2, 9.4, 9.5, 9.6, 9.7, 9.8

**Reference Books**

1 Apostol Tom M., Introduction to Analytic Number Theory, Springer.

2 Baker Alan, A Concise Introduction to the Theory of Numbers, (Cambridge Uni. Press, Cambridge).

3 Hardy G. H. and Wright E. M., An Introduction to Theory of Numbers, (Sixth Edition) Oxford University Press.

## PS01EMTH24: C Programming and Mathematical Algorithms I

Note: 50 Marks (35 marks for external examination and 15 Marks for internal examination) for theory and 50 Marks for practical on computers. External examination will be of two hours for theory and three hours for practical.

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|---|---|
| Unit I | Structure of a C program, the concept of function, preprocessors in C, include statement, function prototype error, comments in C, data types in C, integer family, float family, character family, type casting of variables, arithmetic and relational operators in C, input – output functions, I/O format string, precision of numbers, field width, assignment operators, Mathematical expressions, logical expressions, precedence and associativity of operators, standard library functions, define statement, common programming errors. Arrays in C. |
| Unit II | Branches in C: if, if-else, else-if statements, goto statement, switch statement. Loops: while, do–while, for, break and continue statements, nesting of loops. Statement function definitions. Structures and unions: declaration of structures, accessing structure members, structure initialization, nested structure, array of structures, structure assignment, structure as a function arguments, unions. typedef declaration. |
| Unit III & IV | (List of practical to be performed on computer :) |

| Practical No. | Purpose of the program |
|---|---|
| 1 | Conversion of units: like mile to km. To convert Cartesian coordinates to polar coordinates and vice versa, to convert degree to radian and vice versa, to find simple interest. |
| 2 | To interchange the content of two variables, to find maximum of given 4 numbers, To check given no is odd or even, to check given year is a leap year or not, to find real roots of a quadratic equation, to find all roots of a quadratic equation, to prepare the result of a student. |
| 3 | To find n!, to find $a^n$, to print character table with their ASCII values, To print multiplication table, to find average height of male and female students, to find numbers of students getting first class second class, pass class and fail. |
| 4 | To print numbers for 1 to n such that each line contains m numbers, to check whether given number is prime or not, to check whether given number is perfect or not, to print first k prime numbers, to print all prime numbers $\leq k$, to print first k perfect numbers, to print all perfect numbers $\leq k$, to print Floyd's triangle. |
| 5 | To find sum of digits of a number, to check whether a given number is Armstrong or not (Sum of cubes of digits = the number e,g, $3^3 + 7^3 + 1^3 = 371$), to print a given number in to reverse order of its digits, to check whether given number is palindrome or not, to list first k palindrome numbers, to list all palindrome numbers which are $\leq k$, to find the factors of a natural number. |
| 6 | To find the GCD of two numbers, to find the LCM of two numbers, to find $\sigma(n)$, to find $\tau(n)$, to find $\varphi(n)$, to find $\mu(n)$ (where $\sigma(n)$ is the sum of positive divisors of $n < n$, $\tau(n)$ is number of positive divisors of $n < n$, $\varphi(n)$ is the number of relatively prime numbers to n, |

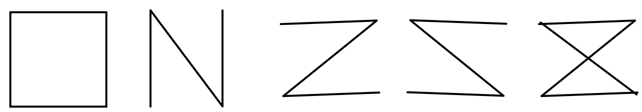
$\mu(n) = 1$ if $n = 1$
$\mu(n) = 0$ if $p^2 | n$ for some prime p
$\mu(n) = (-1)^r$, in $n = p_1 p_2 \ldots p_r$, where $p_1, p_2, \ldots, p_r$ are distinct prime numbers.)

7 To find $e^x$, to find sin(x), to find cos(x), to find sinh(x), to find cosh(x). (Terminate the programs after n terms. Terminate the programs if error is too small)

8 To write programs to count the number of students getting distinction, first class, second class, pass class and fail by using switch statement. Write programs to prepare a frequency distribution table, to prepare a menu generation.

9 To compute approximate solutions of the equation f(x) = 0 by using bisection method, regula falsi method, modified regula falsi method, secant method, Newton's method.

10 Write programs to generate figures like (by using a character):



**Reference Books**


1 Mahapatra P. B., Thinking in C Including object orientated programming with C++, Wheeler Publishing, New Delhi

2 Kernighan B. W. and Ritchie D. M., The C programming Language, Prentice Hall of India Pvt. Ltd., 1990.

3 Rajaraman V., Computer Programming in C, Prentice Hall of India Pvt. Ltd., 1995.