**SARDAR PATEL UNIVERSITY**
**VALLABH VIDYANAGAR**

**SYLLABUS EFFECTIVE FROM: 2017-18**
**Syllabus for M. Phil. (Mathematics)**
**(As per Rules and Regulations for M.Phil. Programme Dec-2016)**

**List of Courses Semester-II**

| | |
|---|---|
| MS02CMTH21 | Research Methodology II |
| MS02EMTH22 | Functional Analysis III |
| MS02EMTH23 | Real Analysis III |
| MS02EMTH24 | Operator Theory |
| MS02EMTH25 | Topological Groups and Abelian Harmonic Analysis |
| MS02EMTH26 | Advanced Banach Algebra Theory |
| MS02EMTH27 | Unbounded Operators in Hilbert Spaces |
| MS02EMTH28 | Advanced Relativity Theory |
| MS02EMTH29 | Riemannian Geometry and Applications of Differential Forms in General Relativity |
| MS02EMTH30 | Problems and Exercises in Mathematics II |
| MS02EMTH31 | Function Algebras |
| MS02EMTH32 | Advanced Graph Theory |

**MS02CMTH21: Research Methodology-II (2 Credits)**

**Unit-I:** Mathematical literature review, choosing a research topic, writing mathematics, presenting mathematics.

**Unit-II:** Modern tools in mathematics: LaTeX – creating a LaTeX file, creating typeset, adding basic information to a document, advanced mathematical type setting, graphics in LaTeX.

**Reference Books**


1 C R Kothari, Reasearch Methodology: Methods and Techniques, New Age International (P) Ltd. (2004).
2 Donald Binder and Martin Erickson, A Student's Guide to Study, Practice and Tools of Modern Mathematics, CRC Press, 2011.

**MS02EMTH22: Functional Analysis III**

**Unit I**: Topological vector spaces, basic properties and examples, classification of topological vector spaces, separation properties, linear functional, finite dimensionality, metrization theorem (without proof), bounded and totally bounded sets, counter examples.

**Unit II**: continuity and boundedness of linear maps, seminorms, local convexity, normability, quotient spaces, Frechet spaces, F-spaces.

**Unit III**: Baire category, Baire's theorem, Banach-Steinhauss theorem, open mapping theorem, closed graph theorem in Frechet spaces, some examples, Hahn-Banach theorems.

**Reference Books**


1 W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.
2 N. Dunford and J. T. Schwarz, Linear operators-I, Interscience, 1973.
3 F. F. Bonsall and J. Duncan, Complete normed algebras, Springer-Verlag.


**MS02EMTH23: Real Analysis III**

**Unit I**: Review of abstract measure space, Measurable functions, simple functions, integration of positive measurable functions, Lebesgue's monotone convergence theorem, Fatou's lemma, integration of complex functions, Lebesgue's dominated convergence theorem.

**Unit II**: Urysohn's lemma and partition of unity (without proof), Riesz representation theorem (without proof), regularity properties of Borel measures, Lusin's theorem, Vitali-Caratheodory theorem, $L^p$-spaces, approximation by continuous functions in $L^p$-spaces.

**Unit III**: Definition of complex measures, total variation measure, positive and negative variations, absolutely continuous measures, mutually singular measures, Lebesgue-Radon-Nikodym theorem, its applications.

**Reference Books**


1 W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.(Chap. 2,3,6,7 & 8).
2 E. Hewitt and K. Stromberg, Real and abstract analysis, Springer Verlag, 1965.


**MS02EMTH24: Operator Theory**

**Unit I:** Review of Hilbert space H, orthogonal complement in H, sesquilinear functional, bounded operator, existence of adjoint operator and its properties, self-adjoint operator and its properties, unitary operator and its properties, Fuglede-Putnam-Rosenblum theorem (i.e., Commutativity Theorem)

**Unit II:** Resulution of the identity E, the algebra $L^{\infty}$ (E), identifying $L^{\infty}$ (E) with a closed subalgebra of BL(H), spectral theorem and its applications, spectral decomposition

**Unit III:** Symbolic calculus for normal operators and applications, invariant subspace problem, eigenvalues of normal operators, positive operators and square roots, polar decomposition and its uniqueness, unitarily equivalent operators, similar operators.

**Reference Books**


1. W. Rudin, Functional Analysis, Tata McGraw Hill Pub. Company, New Delhi, 1973.
2. J. B. Conway, A Course in Operator Theory, Graduate Studies in Mathematics, Volume 21, American Mathematical Society, Rhode Island, 2000.


**MS02EMTH25: Topological Groups and Abelian Harmonic Analysis**

**Unit I**: Topological groups, its basic properties, neighbourhood system, separation axioms, subgroups and their characteristics, locally compact groups, quotient groups.

**Unit II**: Left and right uniformly continuous functions, definition of $(f : \varphi)$, properties of $(f : \varphi)$, properties of the functional $I_\varphi$, existence of left Haar integral (without proof), uniqueness of left Haar integral, Existance of Haar measure on locally compact group, modular function, unimodular groups,

**Unit III**: Characters of LCA groups, dual group $\hat{G}$ of an LCA group, dual groups of compact and discrete abelian groups, Banach algebra $L^1(G)$, its Gelfand space, its semisimplicity, positive definite functions, Separation property of $\hat{G}$ , Pontryagin duality theorem (without proof).

**Reference Books**


1 T. Husain, Introduction to topological groups, W. B. Saunders Co., 1966.
2 R. Larsen, Introduction to Banach Algebras, Marcel Dekker, 1973.
3 W. Rudin, Fourier analysis on groups, Inter science Publishers, 1962.


**MS02EMTH26: Advanced Banach Algebra Theory**

**Unit-I:** Definition and examples of Banach algebras, invertible elements, spectrum and spectral radius of Banach algebra elements, spectral radius theorem, advanced results on spectrum and spectral radius

**Unit-II:** Multiplicative linear functional and its characterizations, Gleason-Kahane-Zelazko Theorem, Automatic continuity, Uniqueness of complete norms, Gelfand theory of commutative Banach algebras, Gelfand spaces of group algebras and Beurling algebras,

**Unit-III:** Functional calculus, holomorphic functional calculus, its basic properties (without proof), some applications of functional calculus, Silov's idempotent theorem and its applications

**Reference Books**


1. E. Kaniuth, A Course in Commutative Banach Algebras, Springer, New York, 2009.
2. R. Larsen, Banach Algebras, Marcell-Dekker, 1973.

**MS02EMTH27: Unbounded Operators in Hilbert Spaces**

**Unit I**: Definition of unbounded operator, its basic properties and examples, adjoint operator and its properties, graph of operators, closed and closable operators, their properties.

**Unit II**: Unitary operators U and V, symmetric operators, self adjoint operators, Cayley transform, its properties and application, deficiency indices.

**Unit III**: Resolution of identity, domain $D_f$, the unbounded operator $\Psi(f)$ for a measurable function f, its properties, Spectral theorem for unbounded normal operators.

**Reference Books**


1 T. Kato, Perturbation Theory for Linear Operators, Springer Verlag.
2 W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.
3 J. Weidmann, Linear Operators in Hilbert Spaces, Springer Verlag.


**MS02EMTH28: Advanced Relativity Theory**

**Unit I**: Killing vectors, maximally symmetric spaces: uniqueness and construction, spherically symmetric homogeneous space-times.
**Unit II**: Algebraic properties of Riemann tensor and Petrov classification.
**Unit III**: The cosmological problem, the Robertson-Walker metric, the red shift measures of distance, red shift versus distance relation, number counts, the standard model, curvature and the future of the universe, the matter dominated era, intergalactic emission and absorption processes.

**Reference Books**


1 Steven Weinberg, Gravitation and Cosmology, John-Wiley and Sons Inc.
2 Misner, Wheeler, Thorne, Gravitation, W. H. Freeman and Co.
3 Banerjee, S. and Benerjee, A, The Special Theory of Relativity, Printice Hall of India.
4 Adler, R. Bazin, M. and Schiffer, M., Introduction to General Relativity (Second Edition) Mcgraw Hill.


**MS02EMTH29: Riemannian Geometry and Applications of Differential Forms in General Relativity**

**Unit I**: Wedge product and exterior differentiation, linearly independent vector fields $e^a$; $a$ = 1,…, n and Ricci rotation coefficients.
**Unit II**: Differential forms, definition of 1-form and 2-forms, frame components of Riemannian curvature tensor $R^a_{bcd}$ .
**Unit III**: Catan's first equation of structure, second equation of structure, curvature 2-forms, examples on computation of tensors in general relativity.

**Reference Books**


1 Differential forms in General Relativity Communication of Dublin Institute of Advanced Studies, Series A No 19(1970).
2 Z. Ahsan, Tensors Mathematics of Differential Geometry and Relativity, PHI Learning Pvt. Ltd., Delhi (2015).


**MS02EMTH30: Problems and Exercises in Mathematics -II**

A student will be required to prepare for Problems and Exercises in any three of the following subjects approved by the Department.

1 Analysis
2 Algebra
3 Topology
4 Functional Analysis
5 Measure Theory
6 Differential Equations
7 Banach Algebras and Operator Theory
8 Classical Mechanics

**MS02EMTH31: Function Algebras**

**Unit I**: Review of commutative Banach algebras, Function algebras, Examples, Gel'fand space of C(X), A(X), P(X) and R(X) for $X \subset \mathbb{C}^n$, Point derivations.

**Unit II**: The dual of C(X), Representing measures, Annihilating measures, Wermer's maximality theorem, Choquet boundary, Silov boundary.

**Unit III**: Peak sets, Antisymmetric sets and Bishop's theorem, Essential sets.

**Reference Books**


1. A. Browder, Introduction to function algebras, W. A. Benjamin, 1969.
2. T. Gamelin, Uniform algebras, Printice Hall, 1969.
3. G. M. Leibowitz, Lectures on complex function algebras, Scoot-Foresman and Company,1970.
4. R. Larsen, Introduction to Banach Algebras, Marcel Dekker, 1973.

**MS02EMTH32: Advanced Graph Theory**

**Unit I: Matrices related with graphs**: Adjacency matrix, Laplacian matrix, Distance matrix etc., properties of these matrices and their spectrum.
**Unit II:Diffeent products of graphs:**Cartesian product, Tensor product etc. of graphs, graph parameters of these products.
**Unit III: Domination Theory and some special graphs:** Domination set, domination no. and their properties, graph parameters of some special graphs.

**Reference Books**


1. C. Godsil and G. Royle, Algebraic Graph Theory.
2. R. Meris, Graph Theory.
3. B. Bollobas, Modern Graph Theory.
4. R. Hammack, W. Emrich and S. Klavzar, Handbook of Product Graphs.
5. Narsingh Deo, Graph Theory with Applications to Engg. & Comp.Sci.