SARDAR PATEL UNIVERSITY
VALLABH VIDYANAGAR

SYLLABUS EFFECTIVE FROM: 2017-18
Syllabus for M. Phil. (Mathematics)

(As per Rules and Regulations for M.Phil. Programme Dec-2016)

The structure is as per the Rules and Regulations for M.Phil. Programme Dec-2016 R.M.Phil.8 of the university. In each semester a student has to take one course of Research Methodology (2 credits) and two courses of advanced level (3 credits each) and earn total 8 credits in a semester. The session work for each course of three credits will comprise of 3 formal lectures per week, each of 1 hour duration. There will be 1 seminar and a problem session of 1 hour duration in each course per week. For the course of Research Methodology, the session work will comprise of 2 formal lectures per week, each of 1 hour duration and 1 seminar and other relevant activity of 1 hour duration per week. Each course will have a weighting of 100 marks (70 marks for University examination + 30 marks for Internal Assessment). Internal Assessment will comprise of 1 internal test of 20 marks and a seminar of 10 marks. Besides these there will be a dissertation consisting of 200 marks to be written under the supervision of recognized M. Phil. supervisor. There will be a viva-voce examination of 100 marks for the dissertation. The dissertation and viva together carry 8 credits. For passing criteria Rules and Regulation for M.Phil Programme (December 2016) of Sardar Patel University should be referred.

List of Courses - Semester-I

MS01CMTH21	Research Methodology I
MS01EMTH22	Functional Analysis III
MS01EMTH23	Real Analysis III
MS01EMTH24	Operator Theory
MS01EMTH25	Topological Groups and Abelian Harmonic Analysis
MS01EMTH26	Advanced Banach Algebra Theory
MS01EMTH27	Unbounded Operators in Hilbert Spaces
MS01EMTH28	Advanced Mechanics
MS01EMTH29	Topics in Relativity Theory
MS01EMTH30	Problems and Exercises in Mathematics I
MS01EMTH31	Function Algebras
MS01EMTH32	Advanced Graph Theory
MS01EMTH33	Advanced Relativity Theory

MS01CMTH21: Research Methodology I (2Credits)

Unit-I: Research as a creative and strategic thinking process, meaning and definition of research, types of research, ethics in research, intellectual property rights, plagiarism. Bibliometric indicators : Impact Factor, H–index of an author, MathSciNet etc.

Unit-II: Modern tools in mathematics:
Mathematica – as a calculator, functions in Mathematica, Graphs in Mathematica, Programming in Mathematica.
MatLab or Octave or SciLab – exploring linear algebra, graphics in MatLab, about programming in MatLab.

Reference Books

1 C R Kothari, Reasearch Methodology: Methods and Techniques, New Age International (P) Ltd. (2004).
2 Donald Binder and Martin Erickson, A Student's Guide to Study, Practice and Tools of Modern Mathematics, CRC Press, 2011.

MS01EMTH22: Functional Analysis III

Unit I: Topological vector spaces, basic properties and examples, classification of topological vector spaces, separation properties, linear functional, finite dimensionality, metrization theorem (without proof), bounded and totally bounded sets, counter examples.

Unit II: continuity and boundedness of linear maps, seminorms, local convexity, normability, quotient spaces, Frechet spaces, F-spaces.

Unit III: Baire category, Baire's theorem, Banach-Steinhauss theorem, open mapping theorem, closed graph theorem in Frechet spaces, some examples, Hahn-Banach theorems.

Reference Books

1 W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.
2 N. Dunford and J. T. Schwarz, Linear operators-I, Interscience, 1973.
3 F. F. Bonsall and J. Duncan, Complete normed algebras, Springer-Verlag.

MS01EMTH23: Real Analysis III

Unit I: Review of abstract measure space, Measurable functions, simple functions, integration of positive measurable functions, Lebesgue's monotone convergence theorem, Fatou's lemma, integration of complex functions, Lebesgue's dominated convergence theorem.

Unit II: Urysohn's lemma and partition of unity (without proof), Riesz representation theorem (without proof), regularity properties of Borel measures, Lusin's theorem, Vitali-Caratheodory theorem, L^p-spaces, approximation by continuous functions in L^p-spaces.

Unit III: Definition of complex measures, total variation measure, positive and negative variations, absolutely continuous measures, mutually singular measures, Lebesgue-Radon-Nikodym theorem, its applications.

Reference Books

1 W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.(Chap. 2,3,6,7 & 8).
2 E. Hewitt and K. Stromberg, Real and abstract analysis, Springer Verlag, 1965.

MS01EMTH24: Operator Theory

Unit I: Review of Hilbert space H, orthogonal complement in H, sesquilinear functional, bounded operator, existence of adjoint operator and its properties, self-adjoint operator and its properties, unitary operator and its properties, Fuglede-Putnam-Rosenblum theorem.

Unit II: Resulution of the identity E, the algebra L^{∞} (E), identifying L^{∞} (E) with a closed subalgebra of BL(H), spectral theorem and its applications, spectral decomposition

Unit III: Symbolic calculus for normal operators and applications, invariant subspace problem, eigenvalues of normal operators, positive operators and square roots, polar decomposition and its uniqueness, unitarily equivalent operators, similar operators.

Reference Books

1. W. Rudin, Functional Analysis, Tata McGraw Hill Pub. Company, New Delhi, 1973.
2. J. B. Conway, A Course in Operator Theory, Graduate Studies in Mathematics, Volume 21, American Mathematical Society, Rhode Island, 2000.

MS01EMTH25: Topological Groups and Abelian Harmonic Analysis

Unit I: Topological groups, its basic properties, neighbourhood system, separation axioms, subgroups and their characteristics, locally compact groups, quotient groups.

Unit II: Left and right uniformly continuous functions, definition of $(f : \varphi)$, properties of $(f : \varphi)$, properties of the functional I_{φ}, existence of left Haar integral (without proof), uniqueness of left Haar integral, Existance of Haar measure on locally compact group, modular function, unimodular groups,

Unit III: Characters of LCA groups, dual group $\hat{G}$ of an LCA group, dual groups of compact and discrete abelian groups, Banach algebra $L^1(G)$, its Gelfand space, its semisimplicity, positive definite functions, Separation property of $\hat{G}$, Pontryagin duality theorem (without proof).

Reference Books

1 T. Husain, Introduction to topological groups, W. B. Saunders Co., 1966.
2 R. Larsen, Introduction to Banach Algebras, Marcel Dekker, 1973.
3 W. Rudin, Fourier analysis on groups, Inter science Publishers, 1962.

MS01EMTH26: Advanced Banach Algebra Theory

Unit-I: Definition and examples of Banach algebras, invertible elements, spectrum and spectral radius of Banach algebra elements, spectral radius theorem, advanced results on spectrum and spectral radius

Unit-II: Multiplicative linear functional and its characterizations, Gleason-Kahane-Zelazko Theorem, Automatic continuity, Uniqueness of complete norms, Gelfand theory of commutative Banach algebras, Gelfand spaces of group algebras and Beurling algebras,

Unit-III: Functional calculus, holomorphic functional calculus, its basic properties (without proof), some applications of functional calculus, Silov's idempotent theorem and its applications

Reference Books

1. E. Kaniuth, A Course in Commutative Banach Algebras, Springer, New York, 2009.
2. R. Larsen, Banach Algebras, Marcell-Dekker, 1973.

MS01EMTH27: Unbounded Operators in Hilbert Spaces

Unit I: Definition of unbounded operator, its basic properties and examples, adjoint operator and its properties, graph of operators, closed and closable operators, their properties.

Unit II: Unitary operators U and V, symmetric operators, self adjoint operators, Cayley transform, its properties and application, deficiency indices.

Unit III: Resolution of identity, domain D_f, the unbounded operator $\Psi(f)$ for a measurable function f, its properties, Spectral theorem for unbounded normal operators.

Reference Books

1. T. Kato, Perturbation Theory for Linear Operators, Springer Verlag.
2. W. Rudin, Functional Analysis, Tata McGraw Hill Ltd, 1974.
3. J. Weidmann, Linear Operators in Hilbert Spaces, Springer Verlag.

MS01EMTH28: Advanced Mechanics

Unit I: Overview of Lagrangian and Hamitonian formulation. Canonical transformation and its role.
Unit II: Transition from discrete to continuous systems, Lagrangian and Hamiltonian formulations for continuous systems, stress-energy tensor.
Unit III: Conservation theorems, Noether's theorem

Reference Books

1 H. Goldstein, Classical Mechanics, Second Edition, Addison-Wesley, 1980.

MS01EMTH29: Topics in Relativity Theory

Unit I: Special theory of Relativity: Michelson-Morley experiments, Lorentz transformation and relativistic mechanics.
Unit II: General Relativity: tensor in general relativity, principle of equivalence, postulates of general relativity.
Unit III: Criteria for Einstein's field equation.

Reference Books

1 Banerjee, S. and Benerjee, A, The Special Theory of Relativity, Printice Hall of India.
2 Adler, R. Bazin, M. and Schier, M., Introduction to General Relativity (Second Edition) Mcgraw Hill.

MS01EMTH30: Problems and Exercises in Mathematics I

A student will be required to prepare for Problems and Exercises in any three of the following subjects approved by the Department.

1 Analysis
2 Algebra
3 Topology
4 Functional Analysis
5 Measure Theory
6 Differential Equations
7 Banach Algebras and Operator Theory
8 Classical Mechanics

MS01EMTH31: Function Algebras

Unit I: Review of commutative Banach algebras, Function algebras, Examples, Gel'fand space of C(X), A(X), P(X) and R(X) for $X \subset \mathbb{C}^n$, Point derivations.

Unit II: The dual of C(X), Representing measures, Annihilating measures, Wermer's maximality theorem, Choquet boundary, Silov boundary.

Unit III: Peak sets, Antisymmetric sets and Bishop's theorem, Essential sets.

Reference Books

1. A. Browder, Introduction to function algebras, W. A. Benjamin, 1969.
2. T. Gamelin, Uniform algebras, Printice Hall, 1969.
3. G. M. Leibowitz, Lectures on complex function algebras, Scoot-Foresman and Company,1970.
4. R. Larsen, Introduction to Banach Algebras, Marcel Dekker, 1973.

MS01EMTH32: Advanced Graph Theory

Unit I: Matrices related with graphs: Adjacency matrix, Laplacian matrix, Distance matrix etc., properties of these matrices and their spectrum.
Unit II: Diffeent products of graphs**:**Cartesian product, Tensor product etc. of graphs, graph parameters of these products.
Unit III: Domination Theory and some special graphs**:** Domination set, domination no. and their properties, graph parameters of some special graphs.

Reference Books

1. C. Godsil and G. Royle, Algebraic Graph Theory.
2. R. Meris, Graph Theory.
3. B. Bollobas, Modern Graph Theory.
4. R. Hammack, W. Emrich and S. Klavzar, Handbook of Product Graphs.
5. Narsingh Deo, Graph Theory with Applications to Engg. & Comp.Sci.

MS02EMTH33: Advanced Relativity Theory

Unit I: Killing vectors, maximally symmetric spaces: uniqueness and construction, spherically symmetric homogeneous space-times.
Unit II: Algebraic properties of Riemann tensor and Petrov classification.
Unit III: The cosmological problem, the Robertson-Walker metric, the red shift measures of distance, red shift versus distance relation, number counts, the standard model, curvature and the future of the universe, the matter dominated era, intergalactic emission and absorption processes.

Reference Books

1 Steven Weinberg, Gravitation and Cosmology, John-Wiley and Sons Inc.
2 Misner, Wheeler, Thorne, Gravitation, W. H. Freeman and Co.

3 Banerjee, S. and Benerjee, A, The Special Theory of Relativity, Printice Hall of India.
4 Adler, R. Bazin, M. and Schiffer, M., Introduction to General Relativity (Second Edition) Mcgraw Hill.