Research Project

(A) Functional Analysis

(A-1) Banach and Topological Algebras

1. Commutative Banach algebras A that admit exactly one uniform norm are being investigated using the enveloping uniform algebra U(A) of A. The relations with Shilov regularity is discovered; and the results are applied to topological dynamical systems, uniform algebras on the polydisc and the ball, the multiplier algebras, as well as convolution Banach algebras of abelian Harmonic Analysis. The supnorm on the Banach algebra C(X) is abstractly characterized.

2. Automatic submultiplicativity of seminorms with square property on a linear associative algebra, as well as automatic continuity of homomorphisms on certain classes of topological algebras are established. An orthogonal basis in a topological algebra is shown to be automatically Schauder.

3. Topological concepts in quasi-topology are explored. A functional representation of certain topological algebras using quasitopology is obtained.

4. A discrepancy in the notion of Banach algebras without order is clarified by exhibiting examples, which have incidentally, also enhanced the classification of two-dimensional linear associative algebras.

5. Three Gelfand-Mazur type theorems are proved. One of these provides a C*-property analogue of Zalar's recent generalizations of the Froelich-Ingelstam-Smiley Theorems. The second illustrates that the assumption in Kaplansky's version of the Gelfand-Mazur Theorem can be weakened in the presence of a C*-norm. The third provides a real analogue of Srinivasan's result.

6. A structural analogy between certain aspects of C*-algebras and uniform Banach algebras (more generally between hermitian Banach star algebras and Banach algebras commutative modulo radical) is revealed leading to a hermiticity analogue of Aupetit's theory of radical and commutativity, UUNP analogues of Barnes' results on UC*NP, involutive analogues of Arens-Goldberg results on multiplicativity factors for seminorms as well as spectral characterizations of the Ptak function and the spectral radius.

7. A completely positive operator valued linear map on a not necessarily unital Banach star algebra admits minimal Stinespring dilation iff it satisfies CP-Schwarz inequality iff it is hermitian satisfying Kadison's Schwarz inequality. This is used to obtain operator-valued analogue of Bochner's Theorem.

8. Two classes of Banach algebras viz. those in which every element is a topological zero divisor and those having power series generators are studied.

9. The celebrated Köthe sequence spaces are investigated as topological algebras with pointwise multiplication; and a locally convex topological algebra A having an orthogonal absolute basis is shown to be isomorphic to a Köthe sequence algebra. This

provides an algebra analogue of the well-known bases theorem due to Pietsch. An abstract Grothendic-Pietsch Nuclearity theorem characterizing the linear topological property of nuclearity in terms of the ring theoretic structure of A is proved. A large collection of examples of topological algebras with orthogonal bases is investigated. A unital dual Banach algebra A having a predual A_* which for the weak star topology is sequentially complete admitting an equicontinuous orthogonal basis is dual algebra isomorphic to $l^¥$; whereas a uB-algebra with an orthogonal basis is isomorphic to c_0.

10. The Arens' algebra $L^w[0, 1]$ is a canonical (counter) example in topological algebra theory. Given a finite measure space (X, S, u), the Arens algebra $L^w(X) = Ç\{L^p(X) : 1 £ p < ¥\}$ is investigated revealing the inter-relation between its topological algebra structure and the measure theoretic structure of X. The Gelfand space is identified with the space of all atoms in X., and linear isometries are shown to be determined by the automorphisms of the measure s-algebra modulo null sets.

11. If the indeterminent X in a Fréchet algebra A of power series is a power series generator for A, then either A is the algebra of all formal power series or is the Beurling Fréchet algebra on the non-negative integers defined by a sequence of weights. The algebra A is an inverse limit of a sequence of Banach algebras of power series iff its topology is defined by a sequence of seminorms satisfying certain closability conditions and equicontinuity conditions due to Loy. A non-Banach uniform Fréchet algebra with a power series generator is a nuclear space. A functional analytic description of the holomorphic function algebras on a simply connected planer domain is obtained.

12. Banach and Fréchet algebras with a Laurent series generator are investigated leading, via the discrete Beurling algebras, to functional analytic characterizations of the holomorphic function algebras on the annulus as well as the $C^¥$-algebra on the unit circle.

(A-2) Locally convex spaces and basis theory

13. Structure of nuclear df-spaces and structure of matrix transformations on nuclear Köthe spaces having generalized bases (namely fully-l(p)-bases) were discussed. The deep-rooted relation of generalized nuclearity with generalized bases was subjected to investigation. The topological aspects of this relation were also studied with the use of sequence space theory.

(A-3) Function Algebras and Function Spaces

14. Necessary and sufficient conditions are given for CR(X) to be a direct sum of two of its subalgebras.

15. Tensor product of Function Algebras, Real Banach algebras were taken up in detail. Certain Banach function algebraic results generalizing the work of Glicksberg and Wada were obtained.

16. On the line of the Bishop and Silov decompositions, several other decompositions have been defined and studied for a function algebra. Certain conditions have been discussed under which some of these or all of these decompositions coincide. Then, this study is generalized for function spaces and real function spaces. It is proved that the Bishop decomposition for function spaces is finest decomposition with the CS1-property. Also, these decompositions are obtained for the tensor product and slice product of function algebras and function spaces.

17. Based upon the idea of anti-symmetric decomposition, several other decompositions for the associated affine function space A(Z) of a function algebra A were introduced and studied. This study is generalized to Function Spaces, Real Function Algebras and Vector Function Algebras. An important property called the (D)-property has been discovered. Various antisymmetric decompositions for vector case are introduced and are compared. For studying weakly prime sets for Real Function Algebra, the concept of an (i)-peak set is introduced and studied. The decompositions in real case are compared with complexification. Also, in case of Function spaces, some of these decompositions are studied for A(Z).

18. In the study of Function algebras and various related properties, it is necessary to have several examples. A construction for obtaining new function algebras out of the given ones is introduced and its point derivations are characterized. Further, the classes like Dirichlet algebras, URM algebras are also studied for this construction.

19. The concept of simultaneous linear extensions (SLE) is well studied for subspaces of C(X). The concept of simultaneous algebraic extension (SAE) had been introduced for function algebras. SAE for vector function algebras and recently characterized SLE for function spaces are studied.

20. A comparative study of known Ascoli-Arzela theorems in various Banach spaces A of functions has led to an uncovering of a general philosophy that compactness in A is characterized by the uniformization of the defining properties of functions in A. This is applied to show that a closed and bounded subset of the Banach spaces of differentiable functions is compact iff it is equidifferentiable.

(B) Operator algebras/operator theory

(B-1) Operator algebras

21. Generalized C*-algebras as a model of certain algebras of unbounded operators is studied. A locally convex *-algebra A is GC*-algebra iff A contains a dense C*-subalgebra continuously embedded in A. Representations of A as extended C*-algebras of Hilbert Space operators are investigated. An irreducible representation of A (more generally a symmetric *-algebra of A) maps elements of A necessarily into bounded operators. Unbounded operator representations of polynomial algebras are investigated using bounded vectors leading to selfadjointness criteria.

22. Pro(jective limits of) C*-algebras, their representation theory and their matricial structure are studied in details leading to a development of the theory of nuclear pro-C*-algebras. Representation theory of a complete locally m-convex *-algebras into unbounded operators is worked out. A Fréchet *-algebras A has a C*-enveloping algebra iff every operator representation of A maps A into bounded operators. The relevance of algebras with C*-enveloping algebras in C*-algebra theory is exhibited.

23. The enveloping s-C*-algebra of smooth Schwartz Fréchet algebra crossed product $S(\tilde{N}, A, a)$ by a smooth action a of $\tilde{N}$ coincides with the s-C*-crossed product $C^*(\tilde{N}, E(A), a)$ of the enveloping algebra $E(A)$ resulting in K-theory isomorphism $K_*(S(\tilde{N}, A, a)) @ K_{*+1}(E(A))$. Spectral invariance of a (not necessarily *-semisimple Fréchet *-algebra A in its enveloping C*-algebra $E(A)$ as well as the associated notion of closure under holomorphic functional calculus are studied, and applied to the differential structure in a C*-algebra A.

24. In the frame work of unbounded non-commutative integration, unbounded operator representations induced by a (quasi) weight on an involutive algebra are studied; and the abstract results are applied to Fourier transforms of measures, tracial weights on O*-algebras, Smooth subalgebras of C*-algebras and to the quasiweights arising in Connes' theory of Non-commutative Geometry.

25. Operator representations of an involutive algebras A into unbounded operators induced by a C*-seminorm defined on a subalgebra of A are studied lending to a characterization of stability of A. A is hereditary C*-spectral iff A is stable and spectral. If A is a pseudo-complete locally convex algebra each element of which is bounded, then A is C*-spectral iff A is stable and spectral iff every algebraically irreducible representation is similar to an algebraically irreducible *-representation. A class of well behaved unbounded operator representations of *-algebras is isolated and is under investigation.

(B-2) Classes of Hilbert Space Operators

26. Operators the operator radii of whose resolvent satisfy certain growth conditions are studied. Spectral properties, Weyl's theorem and various conditions implying normality are investigated for these operators.

27. k-quasihyponormal operators, which arise out of certain norm conditions, are extensively studied. A representation theorem, Fugledge-Putnam theorem and Weyl's theorem are proved. Spectral properties and various conditions implying normality are established. Quasisimilarity relation for such operators is studied and several results of subnormal and hyponormal operators are extended. Local resolvent techniques combined with Fugledge-Putnam theorem are used to unfold the structure of these operators.

28. Operators whose powers are partial isometries are studied and several conditions are obtained under which they become direct sum of isometry and zero.

29. Subnormal operators and d-symmetric operators are studied using C^*-algebra techniques:

1. Certain spectral inclusion property of subnormal operators and their duals are investigated. Subnormal operators are related to the simple unilateral shift; and quasinormal operators are characterized among subnormal operators.
2. d-symmetric operators arise out of certain conditions on the range of derivation operators. Conditions on the commutator deal of associated inclusion algebra are obtained for a d-symmetric operator to be a quasidiagonal or a diagonal operator.
3. Commutators and generalized commutators are studied in the context of Banach algebras.

30. Spectral properties of p-hyponormal operators ($0 < p < 1$) are investigated improving earlier results of Aluthge. A Fugledge-Putnam type Theorem for this class of operators is also developed.
31. The class of 2-isometries introduced by Agler and Stankus is studied. As extension of this, along the lines of partial isometries, partial 2-isometries are introduced; and the differences of their structural properties with those of the partial isometries are highlighted.
32. The class of quasi-isometries is introduced. This generalizes the class of isometries. A comparative study of these two classes is done.
33. The spectral inclusion relation for joint spectrum of an n-tuple of commuting bounded subnormal operators and its minimal n-tuple of commuting normal operators is obtained.
34. Duality relations between joint norms for n-tuple of operators analogous to these between trace norm and operator norm are discussed.

(B-3) Unbounded Hilbert Space Operators:

35. Some results on joint essential spectrum of an n-tuple of unbounded operators analogous to a single bounded operator are proved.
36. A joint spectral theorem for doubly commuting unbounded normal operators is proved by using the techniques of generalized C^*-algebras. This supplements the Gelfand-Naimark approach to the spectral theorem for a bounded normal operator.
37. For an unbounded subnormal operator, the problems of minimal normal extension and spectral inclusion are investigated. Using a modified version of Caley transform, a large class of unbounded subnormal operators is obtained.
38. Unbounded version of Ismagilove multichannel scattering theorem is proved using generalized version of Pearson's theorem.
39. An unbounded operator analogue of Neidhrt-Wollenberg theorem on the converse of Kato-Rosenblum theorem is obtained by using some elementary observations on the structure of co-isometry and self-adjoint operators.
40. Also conditions are investigated under which a closed accretive formally normal operator admits a normal extension, thereby providing analogue of a well-known result on self-adjoint extension of a symmetric operator.

41. Given unbounded operator A r 0 and B m-accretive (respectively self-adjoint), it is shown that under certain commutativity conditions   is m-accretive (respectively self-adjoint).

42. A new class of operators generalizing decomposable operators (due to Colojoara and Foias) called essentially decomposable operators on a Banach space has been introduced and studied in detail. Various examples and applications are studied.

43. Operators of ascent 0 or 1 and operators of descent 0 or 1 are studied. Algebraic, topological and spectral properties and Weyl's theorem are investigated for these operators; and several structure theorems are established for these operators.

44. Banach space operators are studied with respect to their various types of essential spectra and their mapping theorems. Certain norm conditions imposed on them and quasisimilarity relation. Weyl's theorem is examined for functions of hyponormal operators, paranormal operators, isoloid operators, spectral operators and also with respect to a quasisimilar transformation. Conditions for decomposable operators to become spectral operators are obtained. Quasisimilar orbit of a certain class compact operators on a Hilbert space is studied in the context of the hyperinvariant subspace problem.

(C) Harmonic Analysis on abelian groups with weights

45. Let w be a continuous weight function on a locally compact abelian group G. The uniqueness of the uniform norm property on the Beurling algebra $L^1(G, w)$ as well as Banach subalgebras of the weighted measure algebra M(G, w) is investigated. The main tools developed for this consists of the extension of Fourier transform and Fourier-Stieltjes transform to the space H(G, w) of all w-bounded generalized characters on G, weighted analogues of Bochner-Schoenberg-Eberlin Theorem and Wendell's multiplier theorem and the Radon measure algebra on G. An elementary proof of the semisimplicity of $L^1(G, w)$ is obtained.

46. Beurling algebra analogues of the classical theorems of Weiner and Levy on absolutely convergent Fourier series are proved.

47. For a discrete convolution semigroup Banach algebra $l^1(S\ w)$ on an abelian semigroup S, the Gelfand space is identified with w-bounded generalized semi-characters on S, thereby extending classical results of Hewitt and Zuckarman. The radical, weak amenability and uniticity of $l^1(S\ w)$ are characterized in terms of S and w.

48. Given a locally compact abelian group G with a measurable weight w, the Beurling algebra $L^1(G, w)$ admits either exactly one uniform norm or infinitely many uniform norms; and that $L^1(G, w)$ admits exactly one uniform norm if and only if it admits a minimum uniform norm. It is also shown that the set H(G) of all generalized characters on G is LCA group identified with Gelfand space of the convolution algebra $C_C(G)$ with the inductive unique topology. Let M(G, w) consists of locally finite measures m on G

such that mw is a finite regular complex Borel measure. Using Laplace transform it is proved that (i) M(G, w) has unique uniform norm if and only if M(G, w) is regular if and only if L^1(G, w) has unique uniform norm and G is discrete (ii) M(G, w) has a minimum uniform norm if and only if L^1(G, w) has unique uniform norm if and only if M_{00}(G, w) has unique uniform norm if and only if M_{00}(G, w) is regular.

(D) General Relativity and Gravitation

49. **Exact Solutions**: The studies of the Einstein's field equations for gravitational situations in general and in situations with specific symmetries are undertaken. The geometrical features and the physical plausibility of the specific solutions of the respective systems are critically examined. The interplay between the geometrical notions such as conformal structures, embedding properties and the inherent gravitational situations is studied in depth. This study has resulted in obtaining a number of new solutions of Einstein's field equations of physical relevance which include solutions appropriate for describing interior space times of super dense stars in equilibrium, solutions of coupled Einstein-Maxwell equations describing interior space times of charged matter distributions, physically relevant explicit solutions describing string cosmological models on the background of spherically symmetric/cylindrically symmetric/Bianchi space times and also singularity-free solutions of cosmological interests. These studies have also led to developing new procedures for obtaining new solutions of the relativistic field equations, which also include procedures, which start from known solutions serving as seeds for new solutions.

50. **Relativistic Astrophysics:** The studies of problems related with collapse of celestial bodies under the influence of their own gravity are undertaken in ideal and general situations. Various ansatz are developed to study the development of collapse in more realistic situations. Specific ansatz based on geometrical considerations are found to be helpful in describing, models of super dense stars in equilibrium in the absence of definite information about their matter content. Ansatz for constructing models permitting presence of anisotropies in the core regions of super dense configurations have been developed. Of particular mention in these studies are the ansatz given by Vaidya-Tikekar and Tikekar-Thomas respectively assigning spheroidal and pseudo-spheroidal structures for the interior space times of collapsing spherical stellar configurations of matter. These ansatz have been found to be highly instructive in studying collapse accompanied with heat flux, core-envelope models of compact stars in equilibrium and estimates of mass and size of such configurations.

51. **Relativistic Cosmology:** General Relativity Theory is known to predict the beginning of the universe through a singular event of the big bang. It is now realized that GR also permits other possibilities. Following this line of thought the problem of finding singularity-free solutions of Einstein Field Equations describing an ever-existing universe has been undertaken. An independent ansatz for obtaining such models has been worked

out. The effects of the presence of heat flux, scalar fields and strings on the singularity structure of the space-time have been incorporated. Various aspects of spherically symmetric, singularity-free cosmological models of ever-existing universe are being critically scrutinized. These studies (collaborative efforts with N. Dadhich and L. K. Patel) reveal that for a space-time with G_2 symmetry to be a singularity-free it is essential that the kinematic parameter of shear and acceleration should be non-vanishing. Following this a number of spherically symmetric singularity-free imperfect fluid cosmological models could be obtained. The search resulted in obtaining two spherical models of the universe filled with perfect fluid accompanied with heat flux without time like singularity which includes a model of the universe oscillating between two regular states. This approach unlike in Quasi-steady state cosmologies predicts blue shifts without invoking non-conservation of energy or violation of GR.

52. In view of the significance of the Relativistic Kaluza-Klein Theories in five or more dimensional space-times especially in the context of the unification of the fundamental interactions the study of KK theories is undertaken. The solutions of KK field equations describing the space-times of cylindrically symmetric rotating matter, thick domain walls have been obtained. The use of duality technique for obtaining solutions of KK field equations is being critically examined. Higher dimensional versions of the McVittie solution describing a mass particle in FRW universe are obtained.

53. **Geometric Methods in General Relativity:** Einstein's field equations (Efe) in General Relativity form a system of second ordered non-linear partial differential equations and hence very difficult to solve in their original form. Geometrical methods help in simplifying the nature of the field equations. We have simplified Efe for the space-times described by the most general metric represented in the orthogonal form by introducing an eigen value of Weyl conformal tensor in Petrov-Pirani classification. This technique was successfully employed in the case of spherically space-times.

54. **Algebraic Computations in General Relativity:** Efes are expressed in terms of Ricci tensor and its spur. The components of Ricci tensor can be computed by combinations of second order differentials of metrics components and hence the computations are very difficult. We have exploited power of modern computer software MATHEMATICA to carry out this computation.

Mathematical Modeling

55. Diffusion equation was used to investigate the problem of CO_2-concentration on plant canopy (above plant canopy and below plant canopy). Besides, the role of allometry relation in plant growth was found to be of considerable importance. The problem of CO_2-concentration in animal burrows was investigated, through the tools of mathematical modeling.

Tribology

56. After having some run-in, the bearing system develops surface roughness, which in turn, affects the performance of the bearing, mostly adversely. From the industry point of view, the surface roughness effect has been subjected to investigation intensively. Several methods have been proposed to account for the roughness effect. Various types of bearings [Slider Bearings, Squeeze Film Bearings, Bearings working with Magnetic Fluid, Variously layered Porous Squeeze Film Bearings] have been considered. The roughness has been described by a random variable characterized by mean, variance and skewness. It has been found that the magnetic fluid improves the performance of the bearing. Surprisingly it so turns out that the longitudinal surface roughness also enhances performance of the bearing in some of the situations.