Annexure -I

Sardar Patel University

Vallabh Vidyanagar - 388120

Final Report of the work done on the Interdisciplinary Research Project
(Report to be submitted by 15/04/2013)

1. University Reference No. CX-1/Interdisciplinary/DST purse 273

2. Period of report: from 06/06/2012 to 01/04/2013

3. Title of research project An Investigation into some mathematical aspects in Financial Derivatives

4. (a) Name of the Principal Investigator Prof. H. V. Dedania

   (b) Deptt. where work has progressed

   Department of Mathematics, SPU

5. Effective date of starting of the project 06/06/2012

6. Grant approved and expenditure incurred during the period of the report:

a. Total amount approved Rs. 1,00,000=00

b. Total expenditure Rs. NIL

C. Report of the work done: (Please attach five pages in a separate sheets)

i. Brief objective of the project

ii. Work done so far and results achieved (in the following format)

   a) Definition of problem handled
   b) Methodology adapted
   c) Detailed work (including prominent figures/data)
   d) Results and Conclusion
   e) Details of publications (including conference presentation)

iii. Has the progress been according to original plan of work and towards achieving the objective. If not, state reasons. YES

iv. Please indicate the difficulties, if any, experienced in implementing the project No difficulties

v. If project has not been completed, please indicate the reasons: It is completed.

vi. Any other information which would help in evaluation of work done on the project

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Date: 10-04-2013

H. V. Dedania

SIGNATURE OF THE PRINCIPAL INVESTIGATOR

(STAMP)

HEAD OF THE DEPARTMENT

(STAMP)

Head
Department of Mathematics
Sardar Patel University
Vallabh Vidyanagar-388120.

SIGNATURE OF THE CO INVESTIGATOR

Report of the work done

(a) Definition of problem handled:

Black-Scholes-Merton Theory (B-S-M Theory) is a celebrated mathematical theory in finance aimed at explaining the behavior of financial derivatives in Free Capital Market. This three people derived famous option pricing formulas which are known as the B-S-M formulas. In this project we have derived Black-Schloes-Merton formulae for options with different, presumably more realistic, choices of payoff functions.

(b) Methodology adapted:

The mathematical tools used in financial markets include stochastic calculus, stochastic differential equations and Wiener integral. On the other hand, the aspects of Economics in financial markets include stocks, bonds, interest rates and commodities.

The simplest mathematical model used in deriving B-S-M formulas is the following stochastic differential equation.

$$\frac{dS}{S} = \sigma\, dX + \mu\, dt$$

where S is the asset price, σ is the variance rate, μ is the drift rate, and dX is the Wiener process. Using this equation, the following Itô's formula is derived.

$$df = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \right) dt + \frac{\partial f}{\partial S} \sigma S\, dX.$$

By using Itô's formula and theory of partial differential equations, the B-S-M formulas are derived for the plain vanilla payoff function $\max\{ S - K, 0 \}$. Now a days, there are many B-S-M formulas for various payoff functions.

(c) Detailed work:

During this project we have derived the BSM option pricing formulas for the following payoff functions

(1) $C(S,T) = \begin{cases} P(K) - P(S_T), & \text{if } K \geq S_T \\ 0 & otherwise \end{cases}$

(2) $C(S,T) = \begin{cases} P(S_T) - P(K), & \text{if } S_T \geq K \\ 0 & otherwise \end{cases}$

(3) $C(S,T) = \begin{cases} P(S_T) - K, & \text{if } S_T \geq K \\ 0 & otherwise \end{cases}$

(4) $C(S,T) = \begin{cases} K - P(S_T), & \text{if } K \geq S_T \\ 0 & otherwise \end{cases}$

Where P(x) satisfies the following:

(i) $P(x) = a_n x^{p_n} + \ldots + a_2 x^{p_2} + a_1 x^{p_1} + a_0$, $p_n > p_{n-1} > .. > p_1 > 0$, p_i, $a_i \in R$, $n \in N$.
(ii)P(x) is a nondecreasing polynomial on $[0, \infty)$.

(1) and (2) are known as the symmetric fractional polynomial payoff functions and (3) and (4) are known as the asymmetric fractional polynomial payoff functions

(d) Results and Conclusion:

From the deivation of BSM option pricing formulas for the payoff functions above many well known formulas become special cases. For example

1. Plain vanilla option by taking p(x) = x .
2. The standard power option by taking p(x) = x^p
3. The powered options by taking p(x) = $\sum_{i=1}^{n}(-1)^{n-i}\binom{n}{n-i}K^{n-i}x^i$
4. The general power options By taking p(x) = $\sum_{i=1}^{n} a_i x^i$ where $a_i \geq 0$
5. The parabola option by taking p(x) = $a_1 x + a_2 x^2$

(e) Details of publication:

(i) The paper entitled "Option Pricing Formulas for Fractional Polynomial Payoff Functions" has been published recently in the "International Journal of Pure and Applied Sciences", Volume 6 (1), 2013,page 43-48.

(ii) We also presented this paper in the national seminar on "Analysis, Geometry and Applications" held in the Department of Mathematics, Sardar Patel University on 7- 8 March 2013.