Q.1  Answer the following questions by selecting the most appropriate [10] option. Write down the option in your answer book.

(1) If \( y = 7^{5x} \) then \( y'_n = \)______.
   (a) \( 5^n 7^{5x} \) 
   (b) \( 7^n (\log 5)^n 7^{5x} \)
   (c) \( 7^n \cdot 7^{5x} \)
   (d) \( 5^n (\log 7)^n \cdot 7^{5x} \)

(2) If \( y = e^{x} \) then \( y^{16}=\)______.
   (a) 0 
   (b) \( e^x \)
   (c) 1 
   (d) \( e^{-x} \)

(3) If \( y = \cos(3x) \) then \( y^{n}=\)______.
   (a) \( 3^n \cos(3x+\frac{n\pi}{2}) \)
   (b) \( 3^n \cos(3x+\frac{\pi}{2}) \)
   (c) \( 3^n \sin(3x+\frac{n\pi}{2}) \)
   (d) \( 3^n \sin(3x+\frac{\pi}{2}) \)

(4) \( \sqrt{1+(\frac{dy}{dx})^2} = \)______.
   (a) \( \rho \)
   (b) \( \frac{1}{\rho} \)
   (c) \( \frac{ds}{dx} \)
   (d) \( \frac{ds}{dy} \)

(5) For a polar curve, \( \rho = \)______.
   (a) \( \frac{(r^2 + r_1^2)^{\frac{3}{2}}}{r^2 + 2r_1^2 - rr_2} \)
   (b) \( \frac{(r^2 + r_2^2)^{\frac{3}{2}}}{r^2 + 2r_2^2 - rr_2} \)
   (c) \( \frac{(1+r_2^2)^{\frac{3}{2}}}{r_2} \)
   (d) \( \frac{(1+r_1^2)^{\frac{3}{2}}}{r_1} \)

(6) \( \frac{\partial}{\partial x} (\frac{\partial z}{\partial y}) = \)______.
   (a) \( \frac{\partial z}{\partial y} \)
   (b) 0
   (c) \( \frac{\partial^2 x}{\partial y \partial x} \)
   (d) \( \frac{\partial^2 z}{\partial x \partial y} \)
(7) For a function y of x implicitly described by \( f(x, y) = c \), \( \frac{dy}{dx} = \) \[ \text{_____} \].

(a) \( \frac{f_x}{f_y} \)  
(b) \( \frac{f_y}{f_x} \)  
(c) \( -\frac{f_x}{f_y} \)  
(d) \( -\frac{f_y}{f_x} \)

(8) In usual notations, \( \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = \) \[ \text{_____} \].

(a) \( \frac{dz}{dx} \)  
(b) \( \frac{dz}{dt} \)  
(c) \( \frac{dz}{dy} \)  
(d) \( \frac{dy}{dx} \)

(9) The notation \( p = \) \[ \text{_____} \].

(a) \( \frac{\partial y}{\partial x} \)  
(b) \( \frac{\partial x}{\partial y} \)  
(c) \( \frac{dy}{dx} \)  
(d) \( \frac{dx}{dy} \)

(10) The general solution of the differential equation \( y = px + \frac{5}{p} \) is \[ \text{_____} \].

(a) \( y = x + 5 \)  
(b) \( y = cx + \frac{5}{c} \)  
(c) \( cx + \frac{5}{c} = 0 \)  
(d) \( y = cp + \frac{5}{c} \)

Q.2 Write down any answer of Any Ten questions in short. \[20\]

1. If \( y = e^{mx} \), then prove that \( y_n = m^n e^{mx} \).
2. If \( y = \sin(ax+b) \) then find \( y_n \).
3. Find \( \phi \) for the curve \( r = a(1 + \cos \theta) \).
4. Find \( \rho \) for \( r = a \theta \).
5. Find \( \frac{ds}{dx} \) for \( y = a \cosh + \frac{x}{a} \).
6. Find the point of intersection of \( r = a(1 + \cos \theta) \) and \( r = -a \cos \theta \).
7. State theorem on total differential.
8. Define: Homogeneous function
9. State Euler's theorem for function of two variables.
10. Examine whether \( (x^2 - 2xy - y^2)dx - (x + y)^2 dy = 0 \) is exact or not.
11. Define: Exact Differential Equation
12. Solve: \( \sin px \cos y = \cos px \sin y + p \)

Q.3

(a) State and prove Leibnitz's theorem. \[05\]

(b) For \( y = \log(ax + b) \), prove that \( y_n = \frac{(-1)^{n-1}(n-1)!a^n}{(ax + b)^n} \) \[05\]

OR

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Q.3
(a) In usual notations prove that, \( \tan \theta = \frac{r}{d \theta} \). \[05\]
(b) If \( x = \cos(\frac{1}{m} \log y) \) then find \( y_n(0) \). \[05\]

Q.4
(a) Find the length of arc of the parabola \( y^2 = 4ax \) (a>0) measured from the vertex to one extremity of its latus rectum. \[05\]
(b) Find the intrinsic equation of the cardioid \( r=a(1+\cos \theta) \). Hence prove that \( s^2 + 9\rho^2 = 16a^2 \), where \( \rho \) is the radius of curvature at any point of the curve. \[05\]

OR

Q.4
(a) Show that the radius of curvature at any point of the curve \( x = ae^\theta (\cos \theta - \sin \theta), y = ae^\theta (\sin \theta + \cos \theta) \) is twice the perpendicular distance of the tangent at the point from the origin. \[05\]
(b) Show that the intrinsic equation of the curve \( y^3 = ax^2 \) is \( 27s = 8a(\sec^3 \psi - 1) \). \[05\]

Q.5
(a) State and prove Euler's theorem for homogeneous function of three variables. \[05\]
(b) If \( z = f(x, y), x = r \cos \theta, y = \sin \theta \), then prove that \( \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 = \left( \frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2 \). \[05\]

OR

Q.5
(a) Verify Euler's theorem for \( z = x^y \log \left( \frac{y}{x} \right) \) and find \( x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} \).

(b) If \( z = xy f \left( \frac{y}{x} \right) \) and \( z \) is constant, then show that \( f \left( \frac{y}{x} \right) = \frac{x}{y} \left[ y + x \frac{dy}{dx} \right] \). \[05\]

Q.6
Prove that the necessary and sufficient condition for the differential equation \( Mdx + Ndy = 0 \) to be exact is that \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \). \[10\]

OR

Q.6
Solve: \( (p + y + x)(xp + x + y)(p + 2x) = 0 \) \[10\]

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